

Math 421, section 1  
**Entrance “exam”**

Due at the beginning of class, Thursday, September 8, 2005

1. (8) Compute  $\int_1^2 \frac{dx}{x(1-2x)}$  and simplify.
2. (6) If  $u(x, y) = e^{y^2x}$ , what are  $\frac{\partial^5 u}{\partial x^5}$  and  $\frac{\partial^2 u}{\partial y^2}$ ?
3. (10) Suppose  $F(x, y) = x \cos(3x+5y)$ . Define  $A(y)$  by the equation  $A(y) = \int_0^1 F(x, y) dx$  and define  $B(y)$  by the equation  $B(y) = \int_0^1 \frac{\partial F(x, y)}{\partial y} dx$ .
  - a) Find  $A(y)$  by evaluating the integral defining it.
  - b) Find  $B(y)$  by evaluating the integral defining it.
  - c) What is the relationship between  $A(y)$  and  $B(y)$ ?
4. (8) Find a solution to  $y'' - 4y' + 5y = 0$  with  $y(0) = 1$  and  $y'(0) = 2$ . Your solution should be written in terms of real functions.
5. (10) Find a  $2 \times 2$  matrix so that  $(1, 0)$  is an eigenvector for the eigenvalue 2 and  $(1, 1)$  is an eigenvector for the eigenvalue 3.
6. (6) For which points  $(A, B)$  in  $\mathbb{R}^2$  is the matrix  $\begin{pmatrix} 3 & A \\ 2 & B \end{pmatrix}$  invertible? For these pairs  $(A, B)$ , what is the inverse?
7. (8) Suppose  $V(x) = |x|$ , the absolute value function. Let  $C = \int_{-1/2}^{1/2} V(x) (\cos(\pi x))^7 dx$  and  $S = \int_{-1/2}^{1/2} V(x) (\sin(\pi x))^7 dx$ . Is  $S = 0$ ,  $S < 0$ , or  $S > 0$ ? Is  $C = 0$ ,  $C < 0$ , or  $C > 0$ ? Briefly explain your answers.
8. (8) Find all values of  $x$  for which the series  $\sum_{n=5}^{\infty} (x-2)^n$  converges. What is the sum of this series?

**Rules** Please treat this as any other homework assignment. That is, you may consult textbooks or acquaintances or me (!), but the written work you hand in must be your own. An answer alone will not receive full credit – you must show supporting computation or give some explanation or both. I will grade what you hand in as an exam. A passing grade will be at least 75% of the 64 points. Familiarity with all of the material tested here is necessary for success in this course. 4 points will be taken off if the exam is not handed in at the beginning of class, and, for each successive day, a further 4 points will be deducted.