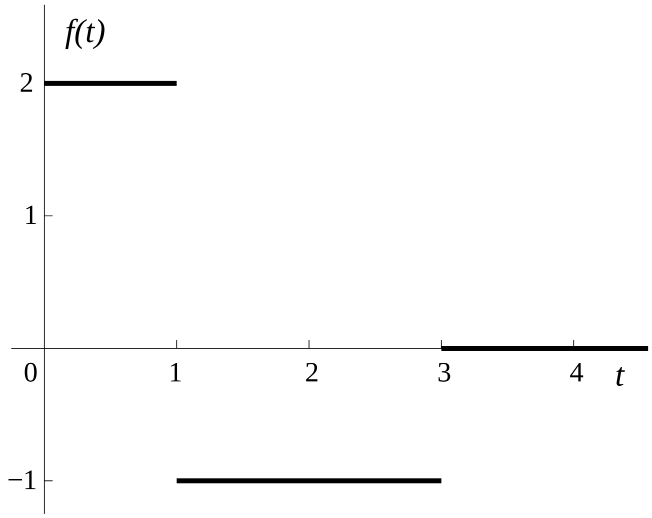
(16) 1. Here is a graph of the function $f(t)$ which is piecewise constant. The values of $f(t)$ are 2 and -1 and 0. $f(t)$ is 0 for all $t > 3$.



a) <u>Use the definition of Laplace transform</u> to find the Laplace transform $F(s)$ of the function $f(t)$.

b) Certainly $\displaystyle\int_0^\infty f(t)\,dt = 0$. Use l'Hopital's rule to verify that $\displaystyle\lim_{s\to 0+} F(s) = 0$. Be sure to indicate why l'Hopital's rule is valid when you use it.

(18) 2. a) Use Laplace transforms to solve the integrodifferential equation $y'(t)+2\displaystyle\int_0^t y(\tau)\,d\tau = 3t$ with initial condition $y(0) = 0$.

b) Check that your answer satisfies the original equation.

$y'(t) =$ ____________________ and $\displaystyle\int_0^t y(\tau)\,d\tau =$ ____________________________ so that

$y'(t) + 2\displaystyle\int_0^t y(\tau)\,d\tau =$ _____________ .

(20) 3. a) Compute $\displaystyle\int_0^5 \left(e^{3t}\right)\big(\mathcal{U}(t-2) + \delta(t-4)\big)\,dt.$

b) Compute the Laplace transform of $\mathcal{U}(t-3)\left(4t + e^{7t}\right)$.

c) Compute the convolution of t and e^{2t}.

(20) 4. a) Solve the initial value problem $y' + y = 2\mathcal{U}(t-1) - \mathcal{U}(t-3)$ with $y(0) = 2$.

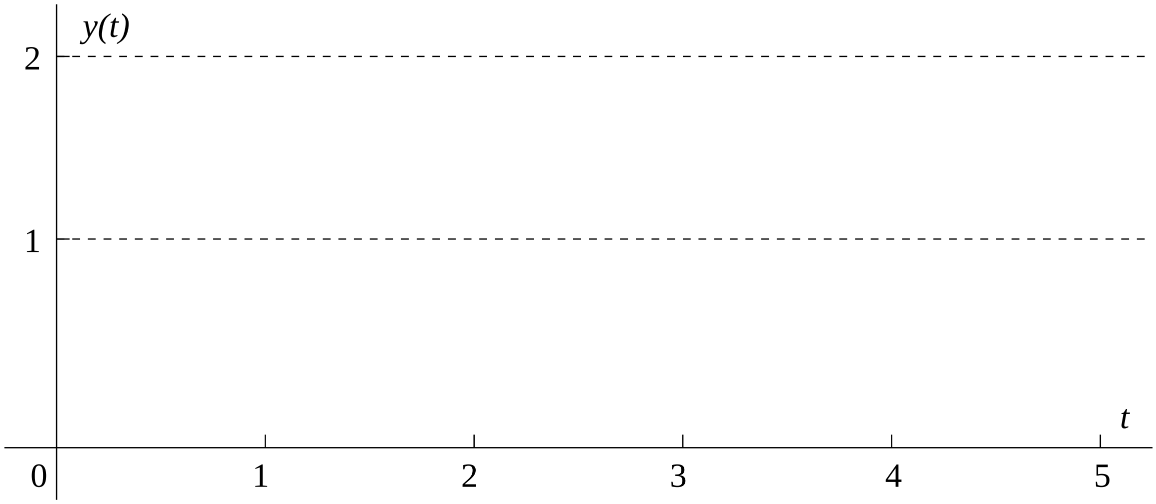
b) Write formulas *without* Heaviside functions for $y(t)$ in the indicated intervals:

If $0 < t < 1$ then $y(t) =$ ______________________________ .

If $1 < t < 3$ then $y(t) =$ ______________________________ .

If $3 < t$ then $y(t) =$ ______________________________ .

c) Graph $y(t)$ as well as you can on the axes below.



d) For which t in the interval $0 < t < 5$ is $y(t)$ continuous?

e) For which t in the interval $0 < t < 5$ is $y(t)$ differentiable?

f) What is $\lim_{t\to\infty} y(t)$?

(14) 5. Find a linear combination of $(2,1,-1,1)$ and $(-1,1,1,2)$ and $(1,1,3,-2)$ which is equal to $(7,1,-11,6)$.

Note You may use one of the RREF's supplied. If you do this, tell which one you use and describe how you use it.

(12) 6. Prove that the three functions t^3 and $t^2(t-1)$ and $t(t-1)(t+1)$ are linearly independent.

First Exam for Math 421, section 1

October 10, 2005

NAME ____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes other than the distributed formula sheet may be used on this exam.
No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	16	
2	18	
3	20	
4	20	
5	14	
6	12	
Total Points Earned:		