(12) 1. Complete the definitions.
a) Suppose A is a $p \times q$ matrix. The *transpose of* A, A^t, is
b) Suppose v_1, v_2, ... and v_t are vectors in $\mathbb{R}^n$. Then v_1, v_2, ... and v_t are *linearly independent* if

(20) 2. Suppose that $A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$.
a) Compute the characteristic polynomial of A.
b) Find the eigenvalues of A.
c) Find a basis of $\mathbb{R}^2$ consisting of eigenvectors of A.
d) Find a diagonal matrix D and an invertible matrix C so that $C^{-1}AC = D$.
e) Find C^{-1}.
f) Compute $Z = AC$.
g) Compute $C^{-1}Z$ using the results of e) and f).
h) Write A as a product of D and C and C^{-1} (in the correct order!) and then use this information to compute $A - A^2 + A^3$.
Note The entries in the matrix which is the answer are 0, 7, 14, and 21.
Please answer the question asked; a direct computation without C will earn no points.

(8) 3. Explain why the matrix $A = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$ *cannot* be diagonalized.

(12) 4. Let A be a non-zero 4×6 matrix.
a) If $\text{rank}(A|B) = 2$ (here B is a 4×1 column matrix), then for what value(s) of $\text{rank}(A)$ is the system $AX = B$, $B \neq 0$, inconsistent? Consistent? Briefly explain your answers.
Comment First state what the possible values of $\text{rank}(A)$ are if $\text{rank}(A|B) = 2$ and why.
b) If $\text{rank}(A) = 3$, then how many parameters does the solution of the system $AX = 0$ have? Briefly explain your answer.

(12) 5. Suppose M is the 5×5 matrix $\begin{pmatrix} a & 0 & 0 & a & a \\ 0 & 0 & b & 0 & 0 \\ c & c & 0 & c & 0 \\ d & 0 & d & d & 0 \\ 0 & e & 0 & e & e \end{pmatrix}$. The determinant of M is a scalar multiple of $abcde$. Compute $\det(M)$.
Reminder Show your work. An answer alone will not receive full credit. Be careful!

(18) 6. In this problem, $f(x)$ is a function whose domain is $[-\pi, \pi]$ and which is defined by the piecewise formula $f(x) = \begin{cases} 0 & \text{if } x < -\frac{\pi}{2} \\ x & \text{if } x \geq -\frac{\pi}{2} \end{cases}$. A graph of $f(x)$ is on the next page.
a) Compute $\displaystyle\int x \sin(nx)\, dx$ if n is not 0.
Comment Yes, this is an indefinite integral. Yes, you should integrate by parts. Yes, you can *guess* the answer, but then you must verify the answer by differentiation.
b) Use the notation for the coefficients of the Fourier series of $f(x)$ on the formula sheet. Find the following Fourier coefficients. The answers for $n > 0$ are sums of rational numbers

(quotients of integers) and rational multiples of $\frac{1}{\pi}$. Use the formula you got in a) and the following result: if $n \neq 0$, $\displaystyle\int x\cos(nx)\,dx = \frac{\cos(nx)}{n^2} + \frac{x\sin(nx)}{n} + C$.

The exact value of the constant term in the Fourier series is ____________________.

The exact value of the first Fourier cosine coefficient, a_1, is ____________________.

The exact value of the second Fourier cosine coefficient, a_2, is ____________________.

The exact value of the first Fourier sine coefficient, b_1, is ____________________.

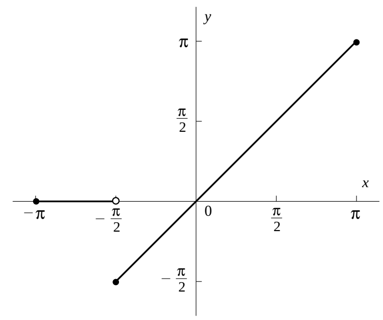
The exact value of the second Fourier sine coefficient, b_2, is ____________________.

c) Suppose $g(x)$ is the partial sum up to the $n=100$ terms in both sine and cosine for the Fourier series of $f(x)$, and $h(x)$ is the sum of the full Fourier series of $f(x)$.

Below are two graphs of $f(x)$ for x in $[-\pi, \pi]$.

Sketch a reasonable approximation to the graph of $y = g(x)$ on the axes to the right.

The student should sketch here the graph of $y = g(x)$, the 100^{th} partial sum of the Fourier series of $f(x)$ on $[-\pi, \pi]$.



Sketch a reasonable approximation to the graph of $y = h(x)$ on the axes to the right.

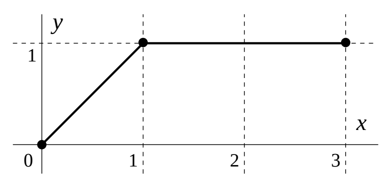
The student should sketch here the graph of $y = h(x)$, the sum of the whole Fourier series of $f(x)$ on $[-\pi, \pi]$.

THE SAME GRAPH.

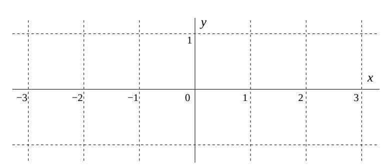
(10) 7. a) Suppose the function $A + Bx^2$ is orthogonal to both the function 1 and the function x on the interval $[0, 1]$. Prove that both A and B must be 0.

b) Find one example of a non-zero function of the form $A + Bx^2$ which is orthogonal to both the function 1 and the function x on the interval $[-1, 1]$.

(8) 8. A graph of $y = f(x)$ is shown to the right. $f(x)$ is a piece-wise linear function and its domain is $[0, 3]$.

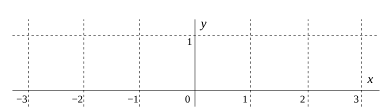


a) Suppose $F(x)$ is the odd extension of $f(x)$ to $[-3, 3]$. Graph $y = F(x)$ on the axes provided below.



Which terms *must* be 0 in the Fourier series of $F(x)$? (Here only an answer is requested, with no explanation needed.) **ANSWER:** ______________________________

b) Suppose $G(x)$ is the even extension of $f(x)$ to $[-3, 3]$. Graph $y = G(x)$ on the axes provided below.



Which terms *must* be 0 in the Fourier series of $G(x)$? (Here only an answer is requested, with no explanation needed.) **ANSWER:** ______________________________

Second Exam for Math 421, section 1

November 17, 2005

NAME ____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes other than the distributed formula sheet may be used on this exam.
No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	12	
2	20	
3	8	
4	12	
5	12	
6	18	
7	10	
8	8	
Total Points Earned:		