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Formulas for exam #2

Fourier series

For $f(x)$ defined in $[-L, L]$, the Fourier series of $f(x)$ is $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{\pi n x}{L}\right)+b_n \sin\left(\frac{\pi n x}{L}\right)$.

Fourier coefficients

$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$; $a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{\pi n x}{L}\right)dx$ and $b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{\pi n x}{L}\right)dx$ for $n > 0$.

Parseval's formula

$\frac{1}{2}a_0^2 + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right) = \frac{1}{L}\int_{-L}^{L} f(x)^2\,dx$.

Orthogonality

If m and n are positive integers, then $\int_{-\pi}^{\pi}\sin(mx)\sin(nx)\,dx = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$ and $\int_{-\pi}^{\pi}\cos(mx)\cos(nx)\,dx = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$ and $\int_{-\pi}^{\pi}\sin(mx)\cos(nx)\,dx = 0$ for all n and m.
If n is a positive integer, then $\int_{-\pi}^{\pi}\cos(0x)\cos(nx)\,dx = \begin{cases} 0 & \text{if } n \neq 0 \\ 2\pi & \text{if } n = 0 \end{cases}$ and $\int_{-\pi}^{\pi}\cos(0x)\sin(nx)\,dx = 0$ for all n and $\int_{-\pi}^{\pi}\sin(0x)\left\{\begin{matrix}\cos(nx)\\ \sin(nx)\end{matrix}\right\}dx = 0$ for all n.