Math 421
Some Fourier series examples

Suppose we define a function $F(x)$ in `Maple`. Below are some `Maple` commands which compute Fourier coefficients and partial sums of the Fourier series. The responses have generally not been given (they are mostly echos of the input).

These `Maple` commands compute the Fourier coefficients of $f(x)$:

```
>h:=n->(1/Pi)*int(F(x)*cos(n*x),x=0..2*Pi);
>g:=n->(1/Pi)*int(F(x)*sin(n*x),x=0..2*Pi);
```

This command computes a partial sum of the Fourier series of $F(x)$:

```
>Q:=N->h(0)/2+sum(h(n)*cos(n*x)+g(n)*sin(n*x),n=1..N);
```

Now let's try these commands. Here is an $F(x)$:

```
>F:=x->(1/10)*x^2;
```

The 1/10 multiplier was chosen so that certain pictures would be easier to understand later. Notice that `Maple` can integrate by parts:

```
>int(x^2*cos(x),x);
      2
     x  sin(x) - 2 sin(x) + 2 x cos(x)
```

Now we can display a partial sum of the Fourier series for $F(x)$:

```
>Q(3);
       2
     2 Pi
     ----- + 2/5 cos(x) - 2/5 Pi sin(x) + 1/10 cos(2 x)
      15
         - 1/5 Pi sin(2 x) + 2/45 cos(3 x) - 2/15 Pi sin(3 x)
```

This is certainly less work than hand computation.. But we also can *see* the relationships between the original function and the partial sums. The command

```
>plot({F(x),Q(3)},x=0..2*Pi,thickness=3,scaling=constrained);
```

will help. This command plots the original function and the third partial sum of the Fourier series of that function on the interval $[0, 2\pi]$. The thickness command draws the graphs a bit heavier or thicker than usual, and the scaling command requires that the horizontal and vertical proportions of the graph be scaled the same. The result is shown to the right.
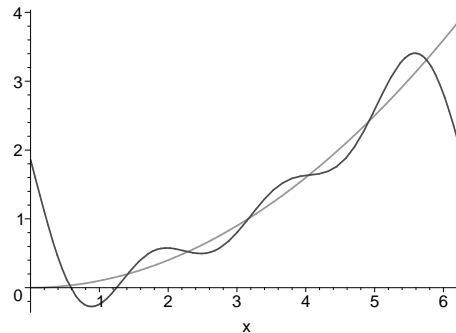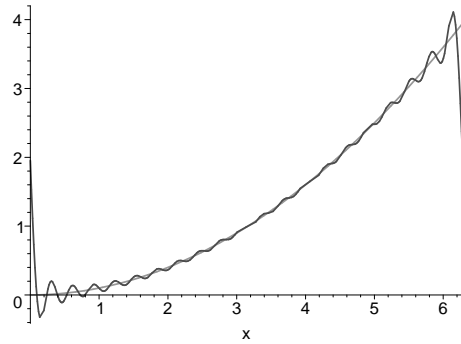

There's more on the other side.

1
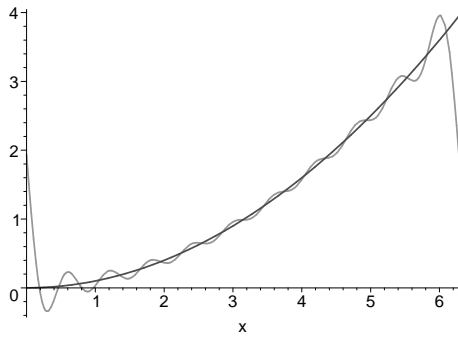
The 10th partial sum and $F(x)$ The 20th partial sum and $F(x)$

Here is something a bit stranger or maybe just more interesting: a piecewise-defined function. This is, of course, the function we met in Laplace transforms called $\mathcal{U}\!\left(t - \frac{\pi}{2}\right)$.
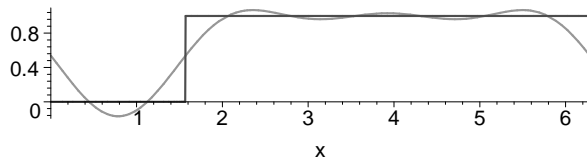
```
>F:=x->piecewise(x<Pi/2,0,1);
```

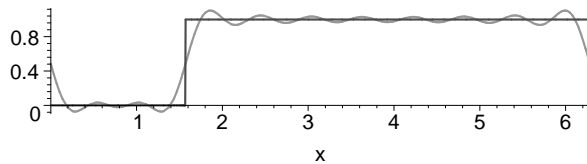Just so you can see it, here is the partial sum up to the $n = 3$ terms of the Fourier series:

```
>Q(3);
        cos(x)    sin(x)    sin(2 x)            cos(3 x)            sin(3 x)
 3/4 -  ------  -  ------ -  --------  + 1/3 --------  - 1/3 --------
          Pi        Pi         Pi               Pi                 Pi
```

And now pictures, showing first the function and the 3rd partial sum:

the function and the 10th partial sum:

and finally, the function and the 20th partial sum:

2