

Problem 2 of **Homework #3** begins

Suppose μ is a compactly supported measure in \mathbb{C} .

I realized that you may not (yet) know what a “compactly supported measure in \mathbb{C} ” is. For the purposes of this problem, please think of μ as the following:

There is a complex linear mapping $T: C(\mathbb{C}) \rightarrow \mathbb{C}$, a compact subset K of \mathbb{C} , and a positive real constant W so that this estimate is true for all $f \in C(\mathbb{C})$:

$$|T(f)| \leq W \sup_{z \in K} \{|f(z)|\}.$$

We will write $T(f)$ as $\int_{\mathbb{C}} f(w) d\mu_w$.

That *should* help you prove that F_μ is holomorphic, and prove the requested limiting property of F_μ . Please tell me if this is correct.

For part b), the compact set is the boundary of the unit circle, and $T(f)$ is the line integral of f over this set. Does the F_μ you discover satisfy the conclusions of a)?

For part c), the compact set is $[0, 1]$, and $T(f)$ is the usual Riemann integral of f over $[0, 1]$. Does the F_μ you discover satisfy the conclusions of a)?

Hint Try to discover F_μ with classical antidifferentiation and “explain” the result.