Final preparation # Math 503 **December 8, 2004**

The purpose of the exam is to assess your knowledge of complex variables and to prepare you for our written qualifying exam. The final exam will be given at 1:10 PM on Friday, December 17, in SEC 220. The exam will have at most 5 questions. At least 2 of the exam questions will be taken from the questions written below. No notes or texts should be used on the exam.

1. Let $\mathcal{P}$ be the set $\{z = n^{3/4} : n \in \mathbb{N}\}$.

a) Prove that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{z - n^{3/4}}$ diverges for all $z \in \mathbb{C} \setminus \mathcal{P}$.

b) Prove that the sum $F(z) = \displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{z - n^{3/4}} + \frac{1}{n^{3/4}} \right)$ converges for all $z \in \mathbb{C} \setminus \mathcal{P}$ and that F is holomorphic in $\mathbb{C} \setminus \mathcal{P}$. What sort of isolated singularity does F have at each $z \in \mathcal{P}$ and what is the residue of F at each $z \in \mathcal{P}$?

2. If f is an entire function which maps every unbounded sequence to an unbounded sequence, then f is a polynomial.

3. Show that there's no f holomorphic in $D(0, 1)$ (the unit disc) with $\displaystyle\lim_{z \to z_0} f(z) = \infty$ for <u>every</u> $z_0 \in \partial D(0, 1)$.

4. Let H be the open upper half-plane: $H = \{z \in \mathbb{C} : \text{Im } z > 0\}$. If $g : H \to H$ is holomorphic and $g(i) = i$, then find and prove some overestimates of $|g(2i)|$ and $|g'(i)|$.

5. Use the Residue Theorem to compute $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1 + x^2 + x^4}\, dx$.