Midterm preparation # Math 503 **November 3, 2004**

The purpose of the exam is to assess your knowledge of complex variables and to prepare you for our written qualifying exam. The midterm exam will be given during the standard class period on Wednesday, November 10. The exam will consists of at most 5 questions. At least 2 of the 5 questions will be taken from the questions written below. No notes or texts should be used on the exam.

1. Suppose that f is entire, and that there is $R > 0$ and a positive integer n so that $|f(z)| \geq R|z|^n$ for all z with $|z| > R$. Show that f is a polynomial, and that the degree of f is at least n.

2. If f is holomorphic in a neighborhood of the closed unit disc, and if $|f(z)| = 1$ when $|z| = 1$, prove that f is a rational function.

Hint One such function is $\dfrac{z-\alpha}{1-\overline{\alpha}z}$ for $\alpha \in D(0,1)$, the open unit disc.

3. Suppose that U is a connected open subset of $\mathbb{C}$ and that S is a discrete subset of U. Prove that S is countable and that $U \backslash S$ is also a connected subset of $\mathbb{C}$.

4. Prove that the annulus $A = \{z \in \mathbb{C} : 1 < |z| < 2\}$ and the punctured unit disc $D(0,1)^* = D(0,1)\backslash\{0\}$ are *not* biholomorphic.

5. Suppose Q is the upper half of the unit disc, as shown, so that $Q = \{z \in D(0,1) : \operatorname{Im} z > 0\}$. Find a biholomorphic mapping of Q with $D(0,1)$. What is the image of the line segment $\{\operatorname{Re} z = 0\} \cap \{0 < \operatorname{Im} z < 1\}$ under this mapping?