

occur.

More formal ~~math~~ description: Suppose  $V$  is a vector space over a field,  $k$ . The projective space,  $P(V)$ , of  $V$  is the set of equivalence classes of  $V^* = V - \{0\}$  under the equivalence relation:  $v, w \in V$ ,  $v \sim w$  if  $\exists \lambda \in k^* = k - \{0\}$  such that  $v = \lambda w$ . Clearly  $v \sim v$  ( $\lambda = 1$ ) and if  $v \sim w$ , then  $w \sim v$  (change  $\lambda$  to  $\frac{1}{\lambda}$ ). For transitivity, multiply the  $\lambda$ 's involved. You can think of  $P(V)$  as the set of directions through the origin in  $V$ .

For us (in the confluence of analysis & geometry) the important fields are  $\mathbb{R}$  or  $\mathbb{C}$ , although it should be noted that applications in algebra or combinatorics frequently consider  $k =$  a finite field.

The most standard examples in topology occur when  $V = \mathbb{R}^{n+1}$  or  $\mathbb{C}^{n+1}$ , and  $P(V)$  is called  $\mathbb{R}P^n$  or  $\mathbb{C}P^n$ , real (resp. complex) projective  $n$ -space.

If  $(z_0, \dots, z_n) \in \mathbb{C}^{n+1}$ , we'll ~~write~~ <sup>let</sup>  $[z_0 : z_1 : \dots : z_n]$  denote the equivalence class of  $(z_0, \dots, z_n)$ . The numbers  $z_0, \dots, z_n$  are called homogeneous coordinates of the equivalence class.

$\mathbb{R}P^n$  and  $\mathbb{C}P^n$  are both topological spaces, with the quotient topology inherited from  $\mathbb{R}^{n+1}$  and  $\mathbb{C}^{n+1}$ .