

Let's be very specific. The case most of interest to us has  $n=1$  and the field being  $\mathbb{C}$ :  $\mathbb{CP}^1$ .

So we look at  $\mathbb{C}^{2*}$ :  $(z_0, z_1)$ , with  $|z_0|^2 + |z_1|^2 \neq 0$ . (Now in fact, the classical algebraic geometers liked the complex numbers more than the reals: it is hard to ~~predict~~<sup>compare</sup>, over  $\mathbb{R}$ , the number of intersections of  $x^2 + y^2 = 1$ ,  $y = 3x$ , and  $x^2 + y^2 = -1$ ,  $y = 3x$ . Indeed.)

This is the complex projective line, with homogeneous coordinates  $[z_0 : z_1]$ . What "is"  $\mathbb{CP}^1$ ? Consider the mapping  $\mathbb{C} \xrightarrow{\sigma} \mathbb{C}^{2*}$  given by  $\sigma(z) = (z, 1)$ .

Then the composition  $\pi \circ \sigma: \mathbb{C} \rightarrow \mathbb{CP}^1$  is an injection, for if  $(z, 1) = \lambda(\tilde{z}, 1)$ , then  $z = \tilde{z}$ . ~~But~~ And since  $\sigma$  is surely continuous, there is a topological "copy" of  $\mathbb{C}$  sitting inside  $\mathbb{CP}^1$ . What does  $\mathbb{CP}^1 - \pi \circ \sigma(\mathbb{C})$  look like? If  $(z_0, z_1) \in \mathbb{C}^{2*}$ , and if  $z_1 \neq 0$ , then  $(z_0, z_1) \sim (\frac{z_0}{z_1}, 1)$ , so  $\pi \circ \sigma(\frac{z_0}{z_1}) = [z_0 : z_1]$ . If  $z_1 = 0$ , then  $(z_0, 0) \sim (1, 0)$  (for  $\lambda = \frac{1}{z_0}$ , since not both homogeneous coordinates can be 0). Thus  $\mathbb{CP}^1 - \pi \circ \sigma(\mathbb{C}) = \{[1:0]\}$ . But, look, all the topological spaces involved are connected or continuous images of connected sets,