

so  $\mathbb{CP}^1$  is connected. Given a sequence  $(w_j) \in \mathbb{C}$ , when does  $[w_j:1] \rightarrow [1:0]$ ? A little thought

should convince you that it should be possible to choose representatives in  $S^3 \subset \mathbb{C}^{2*}$  of  $[w_j:1]$ , call them  $q_j$ , so that  $q_j \rightarrow (1,0) \in S^3$ . Thus the first

coordinates are not 0 for  $j$  large enough. And ( $j$  large)  
 $[w_j:1] = [1; \frac{1}{w_j}] \rightarrow [1:0] \Leftrightarrow$  the modulus  $|w_j|$  grows without bound:  $\forall M > 0 \exists J_M$  s.t.

if  $j \geq J_M$ ,  $|w_j| \geq M$ . The "point"  $[1:0]$  is called " $\infty$ ". And what we have just described is

the one-point compactification of  $\mathbb{C}^1$  (also of  $\mathbb{R}^2$ : confusion avoided!),  $\mathbb{CP}^1$ , as  $\mathbb{C} \cup \{\infty\}$ , with  $\text{ubd}$  of  $\infty$  being complements of compact sets (closed, bdd sets!). But everyone "knows" that  $S^2$  is

such a space, thus  $\mathbb{CP}^1$  is also called the Poincaré sphere. A way of "seeing"  $\pi \circ \sigma: \mathbb{C} \rightarrow \mathbb{CP}^1$

(onto, except ~~for~~ for  $\infty$ ) is to draw