

The mappings $[M]$ form a group (a quotient of $GL_2(\mathbb{C})$, by the appropriate homogeneity), the group of linear fractional transformations, $LF(\mathbb{C})$.

Some nice facts about $LF(\mathbb{C})$ should be recorded.

① We use some language in Remmert (pp. 88-89).

Suppose G is a subgroup of automorphisms of a set, S .

S is homogeneous with respect to G if, $\forall s, \tilde{s} \in S$, $\exists g \in G$ with $g(s) = \tilde{s}$.

Lemma: If $\exists c \in S$ with the orbit of c under G (this is $\{s \in S: \exists g \in G \text{ with } s = g(c)\}$) being all of S , then S is homogeneous.

Proof: Given $s, \tilde{s} \in S$, $\exists g$ & \tilde{g} with $s = g(c)$ and $\tilde{s} = \tilde{g}(c)$. Then $\tilde{g} \circ g^{-1}(s) = \tilde{s}$, so S is homogeneous.

G is also said to act transitively on S . $LF(\mathbb{C})$ acts transitively on $\mathbb{C}P^1$. Indeed, much more is true:

Prop: ~~The~~ The action of $LF(\mathbb{C})$ on $\mathbb{C}P^1$ is triplely transitive. That is given w_1, w_2, w_3 (all distinct) and z_1, z_2, z_3 (all distinct) $\exists [M]$ with $[M]w_j = z_j$, $(1 \leq j \leq 3)$.