Math 503 **Final Exam Preparation** **12/8/2007**

The Math 503 final exam will be given in Hill 525 on **Wednesday, December 19** from 1 to 3 PM (unless you have made other arrangements with me *in advance*). The purpose of the exam is to assess your knowledge of complex variables and to prepare students for our written qualifying exam. Two or three of the following problems will appear on the exam, which will have five or six problems.

1. Suppose that f is continuous on $\mathbb{C}$ and that the function whose value at z is $\big(f(z)\big)^2$, the square of $f(z)$, is holomorphic. Prove that f is holomorphic.

2. What is the automorphism group of $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$? That is, describe all biholomorphic mappings from $\mathbb{C}^*$ to itself. Decide whether or not this group is transitive. If it is not, describe the orbit and stabilizer of 1, and also describe the orbit and stabilizer of 2.

Note The *orbit* of a point for a collection of mappings is the set of image points. The *stabilizer* (also called the *stabilizer subgroup* or the *isotropy subgroup*) of a group of bijections is the subgroup which keeps the indicated object (here, a point) fixed.

3. Prove that if b is a positive real number less than 1, then $\displaystyle\int_0^\infty \frac{x^b}{(1+x)^2}\,dx = \frac{\pi b}{\sin(\pi b)}$.

4. Suppose that u and v are the real and imaginary parts respectively of an entire function f. Find all f such that $u = v^2$.

5. Show that there is no entire function f such that $f\left(\frac{1}{n}\right) = \frac{1}{n+1}$ for all sufficiently large integers, n.

6. The region shown has a line segment and a circular arc as boundary. Find an explicit conformal map taking this region to the unit disc. What is the image in the unit disc of the line segment shown inside the region?