Math 503 **Problem Set 1** **9/7/2007**

Solutions are due at the beginning of class on Friday, September 16, 2007.

Please read chapter 1 of the text and begin reading chapter 2.

1. Suppose f, a real-valued function of a real variable, is defined by $f(x) = \sum_{n=0}^{\infty} \frac{2^n \cos(nx)}{n!}$. Prove that this series converges absolutely and uniformly for all $x \in \mathbb{R}$ and verify that $f(x) = e^{2\cos x}\cos(2\sin x)$. (Assume that absolutely convergent series of complex numbers can be manipulated algebraically as if they were finite sums. This is correct.)

2. a) Here is an exercise in real linear algebra. Consider the vector space $\mathbb{R}^2$ with the usual inner product and usual notion of angle. Two-by-two real matrices act on $\mathbb{R}^2$ by, say, left multiplication (if a vector in $\mathbb{R}^2$ is written as a "column vector"). Find all such matrices which preserve angles between vectors.

Comment One possible approach is to verify that such matrices form a group, and find a transitive subgroup of examples. Use this observation somehow to reduce the problem to finding angle-preserving matrices which fix the vector $(1, 0)$. What are those matrices?

b) Find some way of interpreting the result of a) in terms of complex numbers.

3. Do problem 53 of chapter 1 in the text. Also show how the equation in this problem can be used to verify the conclusion of problem 52 easily.

4. Do problem 55 of chapter 1 in the text.