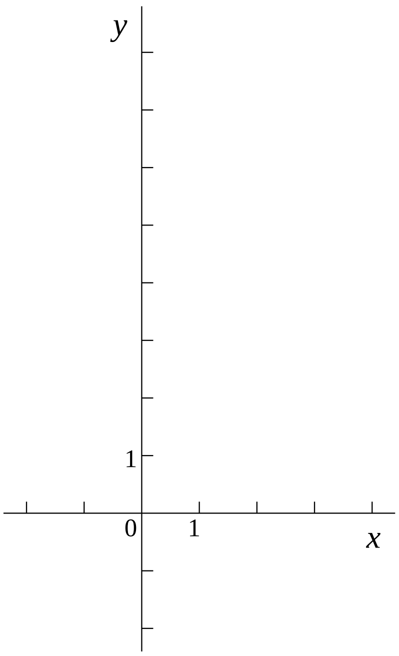
(16) 1. The parabola $y = x^2 - 1$ and the line $y = 2x + 2$ intersect in two points.

- Use algebra to find the coordinates of these points.
- Sketch both curves on the indicated axes.
- Label the curves and label the coordinates of the points where they intersect.

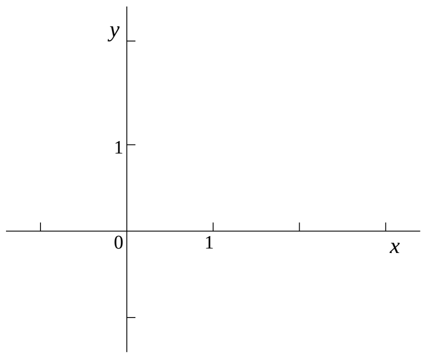


(16) 2. Suppose that the function F is described by

$$F(x) = \begin{cases} x^3, & \text{if } -1 \le x \le 1 \\ \frac{1}{2}x + A, & \text{if } 1 < x \le 3 \end{cases}.$$

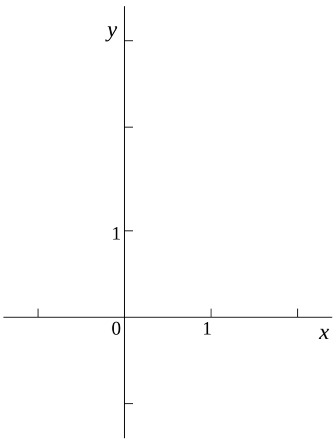
a) Find A so that F is continuous for all numbers in its domain. Explain your answer.

b) Graph $y = F(x)$ for the value of A found in a).



c) Suppose G is the function inverse to F with the value of A found in a). Describe G using one or more formulas: be as precise as you can with G's algebraic description and with its domain.

d) Graph $y = G(x)$.



(20) 3. Evaluate the indicated limits exactly. Give evidence to support your answers.

a) $\lim_{x \to 5} \frac{x-5}{x^2-25}$

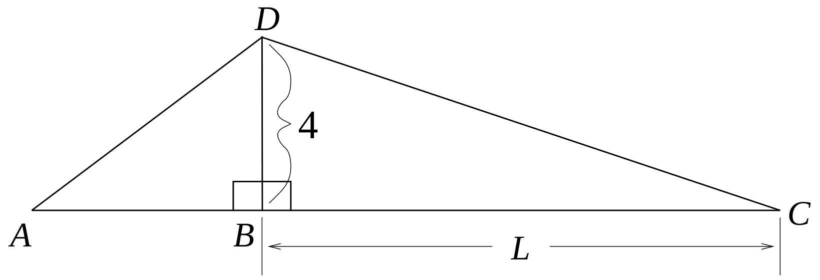
b) $\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4}$

c) $\lim_{x \to 3^-} \frac{x^2+1}{x+2}$

d) $\lim_{x \to 2^-} \frac{x-3}{x-2}$

(10) 4. The following facts should be assumed in all parts of this problem:

The line ABC in the picture is a straight line, and the segment BD is perpendicular to the line, so triangles $\triangle ABD$ and $\triangle BCD$ are right triangles. The length of the line segment BD is 4, and the length of the line segment BC is L.



- Write a formula in terms of an inverse trigonometric function for $\angle DCB$ in radians. Your answer should use 4 and L.

ANSWER: ______________

- Now suppose that $\angle BAD$ is equal to twice $\angle DCB$. What is $\angle BAD$ in radians? Use the answer you have above.

ANSWER: ______________

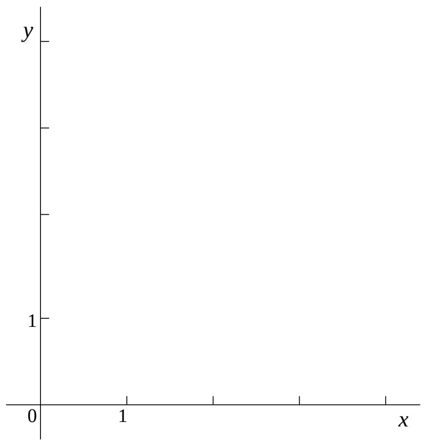
- What is the length of the line segment AD? Use the answer you have above and a trigonometric function.

ANSWER: ______________

(16) 5. a) Write the definition of derivative as a limit and *use this definition* to find the derivative of $f(x) = \dfrac{3}{x}$.

b) Use your answer to a) to find an equation for the line tangent to the curve $y = \dfrac{3}{x}$ when $x = 2$.

c) Sketch the graph of $y = \dfrac{3}{x}$ *and* the line you found in answer to b) on these coordinate axes. Label the curves you draw.



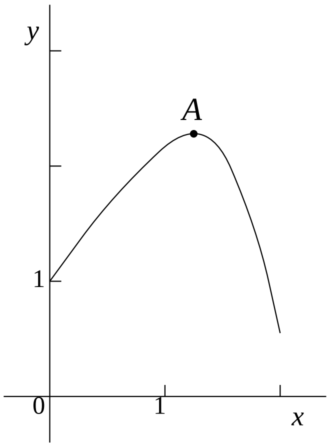
(12) 6. a) If $f(x) = 3x^5 - 8x^2 + 4$, what is $f'(x)$?

b) If $g(x) = \dfrac{3x-2}{2x-3}$, what is $g'(x)$?

c) If $h(x) = x^2 \sin x$, what is $h'(x)$?

d) If h is the function in part c), compute $h(\pi)$ and $h'(\pi)$ exactly, using exact values of any trigonometric functions involved.

(10) 7. The graph of the function $f(x) = 3x - e^x + 2$ in the window $0 \le x \le 2$ and $0 \le y \le 3$ is shown below:



The point A is the highest point on this graph.

a) Draw the tangent line to the graph at the point A on the graph given above. What is the slope of the line you have drawn? Your answer should be very simple.

ANSWER: ________________

b) Use calculus to find the two coordinates of the point A. The coordinates of A should be given exactly, using expressions like "$3\sqrt{5} - 6\ln 2$." Computations with exp and log should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.

A **A**

Exam 1 for Math 135

Sections 16, 17, and 18

October 9, 1997

NAME (*please print*): ______________________________

SIGNATURE: ______________________________

SECTION #: ____________________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use one sheet of notes and any standard calculator without a QWERTY keypad on this exam. You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	16	
2	16	
3	20	
4	10	
5	16	
6	12	
7	10	
Total Points Earned:		

A **A**