Review Problems for the first exam in Math 135

Please note that this collection of 17 problems is definitely longer than any "real" exam. The problems are intended to be representative of the type of problems which may appear on such exams.

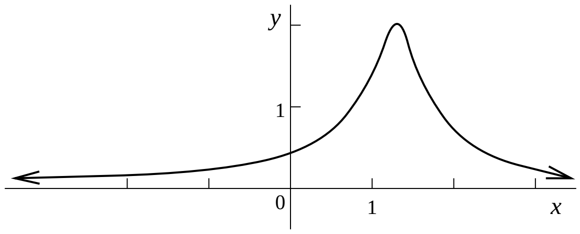
Graphing calculators may be used on exams but calculators and computers with QWERTY keyboards or symbolic differentiation and integration programs are not allowed. Students may bring one sheet of paper containing notes (using both sides!) up to 8.5 by 11 inches in size to any exam.

1. a) Simplify $e^{3\ln\sqrt{x}}$ (give an expression not involving any exponential or logarithmic function).
b) Simplify $\cos(\arcsin\left(\frac{7}{11}\right))$ (give an exact answer).
c) Simplify $\arctan(\tan\left((7\frac{3}{4})\pi\right))$ (give an exact answer).

2. a) Find an equation of the circle C whose center is at $(2, 0)$ and whose radius is 3.
b) Find an equation of the straight line L which goes through the points $(3, 0)$ and $(0, 6)$.
c) Find the coordinates of the points where the straight line L and the circle C intersect. Sketch the line and the circle on the same coordinate axes.

3. Suppose we know that $Q(10^{10}) = 7$, $Q(10^{100}) = 9$, and that $Q(x)$ is a linear function of logarithm: that is, $Q(x) = A(\ln x) + B$. What are the constants A and B?

4. Here is a graph of $y = f(x)$, a function whose domain is all numbers and whose range is $0 < y \le 2$. Sketch as well as you can graphs of the functions $F(x) = \dfrac{1}{f(x)}$ and $G(x) = 3 + 2^{f(x)}$. Specify the domain and ranges of each of these functions as precisely as you can.



Graph of $y=f(x)$

5. A function Z, defined for $x \ge 0$, is given by $Z(x) = \begin{cases} 2x & \text{if } 0 \le x \le 3 \\ x+3 & \text{if } 3 < x \end{cases}$.

a) Sketch the graph of $y = Z(x)$.
b) Compute $Z(2)$ and $Z(4)$.
c) Suppose the function inverse to Z is called N. What is $N(8)$? What is $N(2)$?
d) Sketch the graph of $y = N(x)$.

6. Suppose $p = 100 + 3\sqrt{n}$ represents the production cost, p, in dollars of n boxes of detergent when n satisfies the restriction $1 \le n \le 100$. Solve the equation for n as a function of p. What is the domain of this function of p? Write a phrase which describes what the formula you have obtained represents.

7. Use algebraic methods developed in this course to find the following limits, if they exist. Give evidence supporting your answer.

$$\lim_{x\to 2}\frac{x-2}{x^2+4}\,;\quad \lim_{x\to 2}\frac{x-2}{x^2-4}\,;\quad \lim_{x\to 4}\frac{x-4}{\sqrt{x}+2}\,;\quad \lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}.$$

8. Find the value of these limits, if they exist:

$$\lim_{x\to 0}\frac{1}{x}\,;\quad \lim_{x\to 0^-}\left(\frac{1}{\arctan x}\right);\quad \lim_{x\to 0^+}\arctan\left(\frac{1}{x}\right);\quad \lim_{x\to 0}\arctan\left(\frac{1}{x^2}\right)\,;\quad \lim_{x\to 0}\left(\arctan\frac{1}{x}\right)^2.$$

9. Write an equation of the line tangent to the graph of the function $f(x) = 3x^2 + 4$ when $x = 1$.

10. Use the definition of derivative and familiar properties of limits to find the derivative of $f(x) = \dfrac{3}{x}$.

11. Compute the derivative of these functions:

$$x^3 - 3x + 17;\quad e^x\sin x;\quad \frac{2x+3}{x^2+1};\quad 3e^x + \frac{x}{2+5\cos x}.$$

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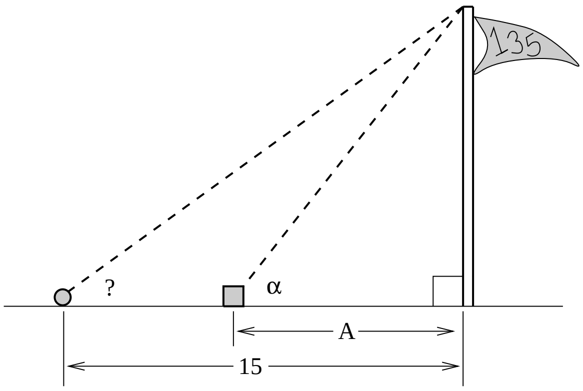
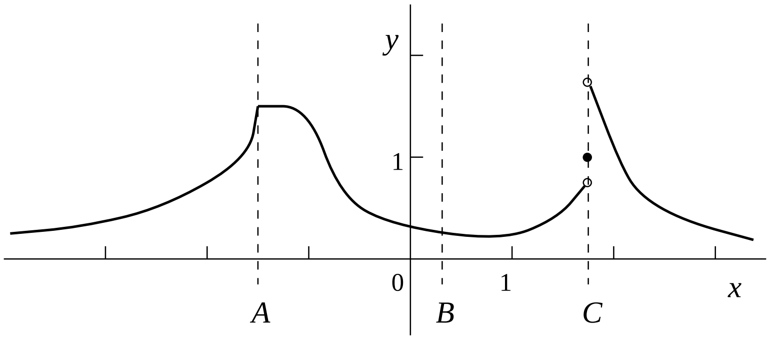
12. A flag pole which is 10 feet tall is placed vertically on flat ground.

a) If a ball on the ground is 15 feet away from the base of the flag pole, what is the angle seen at the ball from the base of the flag pole to the top of the flag pole?
b) If a box on the ground is A feet from the base of the flag pole, write an equation for the angle α seen at the box from the base of the flag pole to the top of the flag pole in terms of A.



13. Let $f(x) = \begin{cases} x^2+2, & x<0 \\ x+A, & 0 \le x < 2 \\ x^2+1, & x \ge 2, \end{cases}$ where A is a number to be determined.
a) Find A such that for this particular A, $f(x)$ is continuous at $x=0$.
b) For the A you obtained in part a), is f also continuous at $x=2$? Explain your answer clearly.

14. Consider the function g whose graph is shown below:



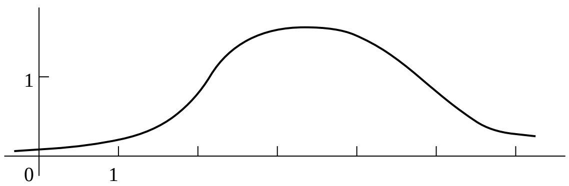
Graph of $y=g(x)$

Answer the following questions as precisely as possible for each of the values $x=A$, $x=B$ and $x=C$. Give reasons for your answers.

Is g continuous at this value of x? Is g differentiable at this value of x?

15. Draw the graph of a function h which has *all* of the following properties:
The domain of h is $0 < x < 3$ and $3 < x < 5$; h is continuous everywhere in its domain; $\lim_{x\to 0^+} h(x) = +\infty$ and $\lim_{x\to 3^-} h(x) = +\infty$ and $\lim_{x\to 3^+} h(x) = -\infty$ and $\lim_{x\to 5^-} h(x) = 1$; h is differentiable everywhere in its domain, and the only x's for which $h'(x)=0$ are 2 and 4; $h(2)=1$ and $h(4)=2$.
Write the equations of any vertical asymptotes of this graph.

16. For each of the following statements, use this graph to find a time interval in which the statement is true. Be sure to explain your assertions.
i) Enzyme concentration is increasing, but the rate of increase is slowing.
ii) Enzyme concentration is increasing and the rate of increase is also increasing.



Enzyme concentration as a function of time in seconds

17. Suppose that f and g are differentiable functions, and the following information is known:

$$f(1)=2; \quad f'(1)=-3; \quad g(1)=5; \quad g'(1)=7.$$

If $F(x)=f(x)g(x)$ and $G(x)=\dfrac{f(x)}{g(x)+x^2}$, compute $F(1)$, $F'(1)$, $G(1)$, and $G'(1)$.

On the web at `http://math.rutgers.edu/~greenfie/currentcourses/math135/math135_index.html`.