Review Problems for the second exam in Math 135

Please note that this collection of 16 problems is definitely longer than any "real" exam. The problems are intended to be representative of the type of problems which may appear on such exams. Many routine review problems are available in the review sections of the text (for example, the *Review Exercises for Chapter 3* appear on pages 256 through 258).

Graphing calculators may be used on exams but calculators and computers with QWERTY keyboards or symbolic differentiation and integration programs are not allowed. Students may bring one sheet of paper containing notes (using both sides!) up to 8.5 by 11 inches in size to any exam.

1. Find $\dfrac{dy}{dx}$ when

a) $y = e^{3\cos 2x}$ b) $y = \ln(x^3 + 5x + 9)$ c) $y = \sin\left(\dfrac{x+1}{x^2+2}\right)$ d) $x^2 + 5xy = e^y + y$

2. Suppose f and g are differentiable functions such that $f(g(x)) = 8x^2$ for all real numbers x. Assume that $f(2) = 7$, $g(2) = 4$, and $f'(4) = 2$. What is $g'(2)$?

3. At a certain time, the length of a rectangle is 3 feet and its width is 5 feet. At that time, the length is *increasing* at .4 feet per second, and the width is *decreasing* at .5 feet per second.

a) At that time, how fast is the **area** of the rectangle changing? Is the area increasing or decreasing?

b) At that time, how fast is the **diagonal** of the rectangle changing? Is the diagonal increasing or decreasing?

4. Let $f(x) = e^{x-x^2}$

a) Graph $y = f(x)$ in the window $-2 \le x \le 2$ and $0 \le y \le 2$.

b) Calculate $f'(x)$. Find the roots of $f'(x) = 0$ algebraically. Where is $f'(x)$ positive and where is it negative? Explain how your results apply to the graph of $y = f(x)$.

c) Calculate $f''(x)$. Find the roots of $f''(x) = 0$ algebraically. Where is $f''(x)$ positive and where is it negative? Explain how your results apply to the graph of $y = f(x)$.

5. Suppose $f(x)$ is a differentiable function and you know the following:

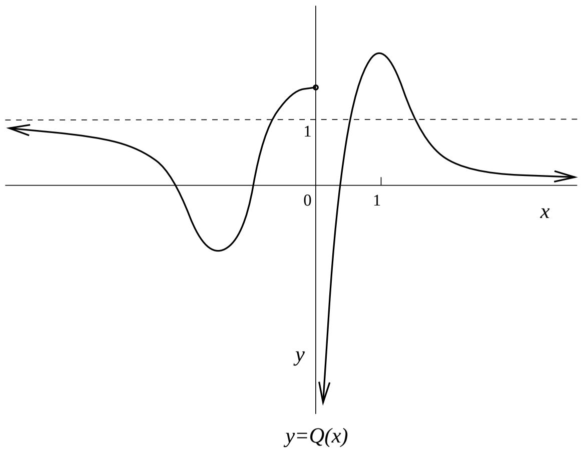
$$f(1) = 10 \text{ and } -1 \le f'(x) \le 2 \text{ for } 1 \le x \le 4.$$

a) Is it possible that $f(4) = 15$? If so, give an example of such a function; if not, give a reason.

b) Is it possible that $f(4) = 3$? If so, give an example of such a function; if not, give a reason.

6. Here is a graph of $y = Q(x)$ which you should use to answer the following questions <u>as well as you can</u>. Some of your answers may have to be approximate. Note that the domain of Q is *all numbers*.

Where is Q continuous? Where is Q differentiable? What are Q's critical numbers? What are the equations of the vertical asymptotes of Q, if any? What are the equations of the horizontal asymptotes of Q, if any? What are the open intervals in which Q is {in/de}creasing? Locate and describe any relative max/min of Q. In what intervals is Q concave {up/down}? Locate any points of inflection of Q. Find these limits, if they exist: $\lim_{x\to-\infty} Q(x)$, $\lim_{x\to 0^-} Q(x)$, $\lim_{x\to 0^+} Q(x)$, and $\lim_{x\to+\infty} Q(x)$.



7. What is the largest rectangle with sides parallel to the coordinate axes that can be enclosed between the $x-$axis and the curve $y = 4 - x^2$?

8. A circular puddle on a flat concrete floor is spreading so that at a certain time, the radius is increasing at .3 inches per second. If the area of the puddle at that time is 25 square inches, how fast is the area changing?

9. Consider the curve defined by the equation $5x^2 + 2xy + 2y^2 = 7$.

a) Find $\dfrac{dy}{dx}$.

b) Which points on the curve have horizontal tangent lines?

c) Which points on the curve have vertical tangent lines?

d) It is known that the points satisfying this equation form a tilted ellipse. Specify a box which contains this ellipse.*

10. Write algebraic restrictions on the appropriate derivatives which mean the same as each of the following separate statements (you will need to be specific about the function and the derivatives!). In each case, sketch a graph of a function having the indicated behavior.

a) The rate of enzyme activity is increasing, but the rate of increase is slowing.

b) The rate of increase of enzyme activity continues to decrease, but more slowly than before.

11. Two non-negative numbers have sum equal to a positive number, P. How should the numbers be selected so that the product of the cube of one times the square of the other is largest?

12. a) Find the maximum and minimum values of the function $Q(x) = x^3 + (x-4)^2$ on the interval $[-4, 4]$.

b) Find the maximum and minimum values of the function $Q(x) = x^3 + (x-4)^2$ on the interval $[0, 4]$.

13. Find the range of $V(x) = e^{-2x} + e^{3x}$.

Note Your answer should *not* be approximate, but should be written in terms of values of well-known functions. Expressions involving ln and exp should be simplified as much as possible.

14. Sketch a graph of a function $W(x)$ which has all of the following properties:

- The domain of W is all numbers except 1 and W is differentiable everywhere in its domain.
- The only root of $W(x) = 0$ is at 0.
- The only roots of $W'(x) = 0$ are at -1 and 2, and $W(-1) = 2$ and $W(2) = 1$.
- The only root of $W''(x) = 0$ is at -2 and $W(-2) = 1$.
- $\lim_{x\to-\infty} W(x) = 0$, $\lim_{x\to 1^-} W(x) = -\infty$, $\lim_{x\to 1+} W(x) = \infty$, and $\lim_{x\to\infty} W(x) = \infty$.

Indicate carefully any relative extrema, any inflection points, and any horizontal or vertical asymptotes.

15. Find $\lim_{x\to\infty}$ and $\lim_{x\to-\infty}$ for the following functions:

a) $\dfrac{e^{2x}}{3 + e^{4x}}$ b) $\dfrac{e^{4x}}{3 + e^{2x}}$ c) $\dfrac{x^3 + 1}{x^2 + x + 1}$ d) $\dfrac{x + 1}{\sqrt{x^2 + 2x + 3}}$

16. An open-topped box has sides costing \$10 per square foot and a square bottom costing \$20 per square foot. What is the cheapest cost for such a box whose volume is 10 cubic feet?

On the web at `http://math.rutgers.edu/~greenfie/currentcourses/math135/math135_index.html`.

* Hint: