(14) 1. Use the method of partial fractions to verify that

$$\int_0^1 \frac{1}{(x+1)(x^2+1)}\, dx = \frac{1}{4}\ln 2 + \frac{1}{8}\pi$$

(16) 2. a) The base of a solid is the region bounded by the parabola $y = x^2$ and the line $y = 4$. Cross-sections of the solid by planes perpendicular to the $x-$axis are squares. Find the volume of the solid.

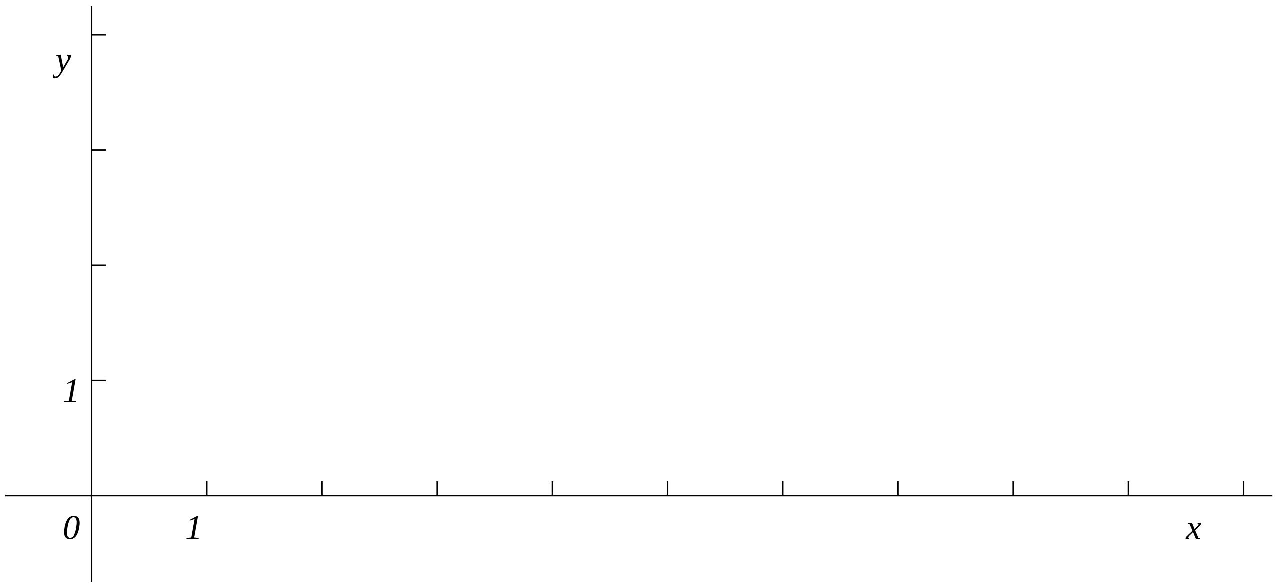
b) The base of a solid is the region bounded by the parabola $y = x^2$ and the line $y = 4$. Cross-sections of the solid by planes perpendicular to the $y-$axis are squares. Find the volume of the solid.

(12) 3. Use integration by parts followed by a substitution (or a substitution followed by integration by parts) to verify that

$$\int_0^1 e^{\sqrt{x}}\, dx = 2$$

(16) 4. Suppose $\mathcal{R}$ is the region in the first quadrant bounded by the two curves $y = 4e^{-2x}$ and $y = 4e^{-3x}$.

a) Sketch that part of the region between $x = 0$ and $x = 10$ on the axes given.

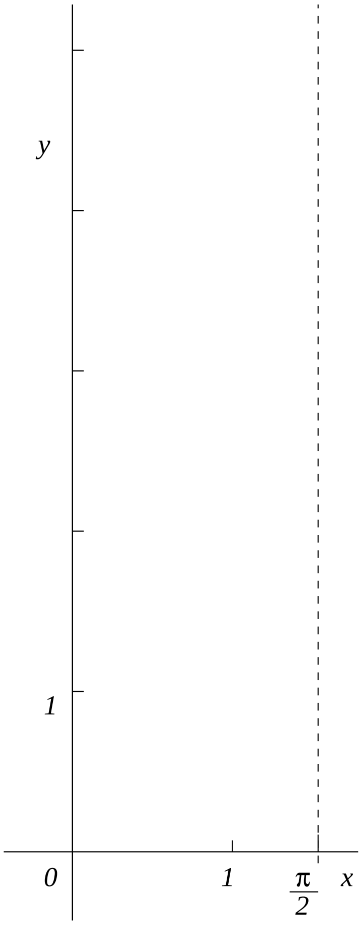


b) Compute the area of the whole region $\mathcal{R}$ (out to ∞) if it is finite.

(16) 5. Suppose $\mathcal{S}$ is the three-sided region in the first quadrant bounded by the $y-$axis and the two curves $y = \tan x$ and $y = \sec x$.

a) Sketch that part of the region between $y = 0$ and $y = 5$ on the axes given.

b) Compute the area of the whole region $\mathcal{S}$ (up to ∞) if it is finite.



(12) 6. a) Suppose m and n are positive integers. Find a reduction formula for

$$\int x^m (\ln x)^n \, dx$$

(Here the object is to reduce n, since if we can push n to 0 we'll just have a polynomial to integrate, which is easy.)

b) Use the formula obtained in a) to compute

$$\int x^{20} (\ln x)^2 \, dx$$

(14) 7. This problem analyzes the computation needed to estimate the definite integral

$$\int_0^1 \frac{1}{9} \sin\left(x^3\right) dx$$

a) Find n (the number of subdivisions) so that the Trapezoidal Rule estimate will be within 10^{-6} of the true value of the definite integral. (You may use the error bound $\dfrac{K(b-a)^3}{12n^2}$ where K is an overestimate of the magnitude of the second derivative.)

DO NOT COMPUTE THE TRAPEZOIDAL RULE ESTIMATE.

b) Find n (the number of subdivisions) so that the Simpson's Rule estimate will be within 10^{-6} of the true value of the definite integral. (You may use the error bound $\dfrac{J(b-a)^5}{180n^4}$ where J is an overestimate of the magnitude of the fourth derivative.)

DO NOT COMPUTE THE SIMPSON'S RULE ESTIMATE.

EXAM 1 for MATH 192:03

October 10, 1996

NAME (*please print*): ____________________

SIGNATURE: ____________________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use one sheet of notes and any standard calculator without a QWERTY keypad on this exam. You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	14	
2	16	
3	12	
4	16	
5	16	
6	12	
7	14	
Total Points Earned:		