(12) 1. Use Taylor's Theorem to explain how to compute $\cos(-2)$ to accuracy of $\pm .005$; you should write down a sum which is the desired approximation, and be sure to specify how you know that the error is within the desired tolerance.

(12) 2. What polynomial will approximate e^x with an error of $\pm .01$ on the interval $\left[-\frac{1}{5}, \frac{1}{5}\right]$? Be sure to explain how you know that your answer is correct.

(10) 3. What is

$$\lim_{x\to 0} \frac{\arctan\left(x^2\right) - (\arctan x)^2}{x^4}$$

Don't use l'Hôpital's rule four times! Please don't.

(10) 4. Write $\displaystyle\int_0^2 e^{-x^2}\,dx$ as an infinite series using the Taylor series for the exponential function. Your answer should be in summation notation; indicate the general term as clearly as you can.

(10) 5. For which positive integers k does the series

$$\sum_{n=1}^{\infty} \frac{(n!)^k}{(kn)!}$$

converge?

Comment: you may begin by considering a few "low" special cases like $k = 1$ and $k = 2$, but try as well as you can to give your reasoning in general.

(10) 6. Show that $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ diverges. Also find a specific partial sum $S_N = \displaystyle\sum_{n=2}^{N} \frac{1}{n \ln n}$ with $S_N > 1{,}000$. Be sure to explain how you know that your answer is correct.

(10) 7. Show that $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ converges. Also find a specific partial sum $S_N = \displaystyle\sum_{n=2}^{N} \frac{(-1)^n}{n \ln n}$ so that S_N is within $\dfrac{1}{1{,}000}$ of the sum of the whole series. Be sure to explain how you know that your answer is correct.

(14) 8. Consider the power series $\displaystyle\sum_{n=1}^{\infty} (n^2 + 3n + 7)x^n$.

a) What is the radius of convergence of this series?

b) Inside its radius of convergence, the sum of this series is a rational function (a quotient of polynomials). What is this function?

Comment: you may want to break up the series into several pieces and handle each piece separately. The answer is not necessarily "pretty".

(12) 9. True or false? If false, give an example to show that the implication is not true. If true, briefly explain why.

a) Suppose that all of the a_n's are positive. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \frac{1}{a_n}$ diverges.

b) Suppose that all of the a_n's are positive. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} \frac{1}{a_n}$ converges.

EXAM 2 for MATH 192:03

November 21, 1996

NAME (*please print*): ______________________________

SIGNATURE: ______________________________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use <u>one</u> sheet of notes and any standard calculator without a QWERTY keypad on this exam. You may use <u>no</u> other materials.

Problem Number	Possible Points	Points Earned:
1	12	
2	12	
3	10	
4	10	
5	10	
6	10	
7	10	
8	14	
9	12	
Total Points Earned:		