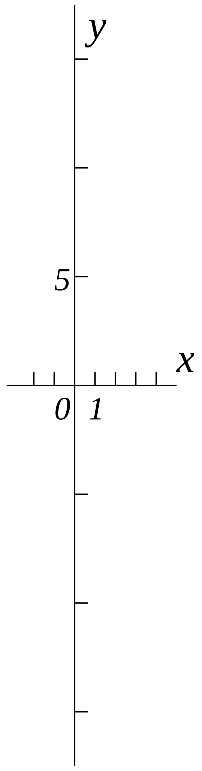
(16) 1. a) Sketch the region which is enclosed by the curves $y = x^2$ and $y = -x^2 + 4x + 6$. Label the curves.



b) Set up and compute a definite integral for the area of the region sketched in a). Compute this integral using calculus.

c) Set up a definite integral for the volume obtained when the region sketched in a) is revolved about the $x-$axis. Describe the method you are using. Do **not** evaluate the integral.

(20) 2. a) What is the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{3^n}{n^2} x^n$?

b) Solve the differential equation $y' = 3 - y$ subject to the initial condition $y(0) = 5$. (Please write y explicitly as a function of x.)

(20) 3. The *sine-integral* function, $\mathrm{Si}(x)$, is defined by

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\, dt$$

a) Use Taylor series for the sine function to show that the function inside the integral sign has nice behavior near 0.

b) Use your answer to a) to find a power series expression for $\mathrm{Si}(x)$.

c) `Maple` reports that $\mathrm{Si}(.5)$ is approximately .493107. Describe how you can check `Maple`'s result, and then use your calculator to get this answer.

(20) 4. `Maple` is *not* able to find an antiderivative of $\left(1+x^3\right)^{5/2}$. It does report the numerical approximation

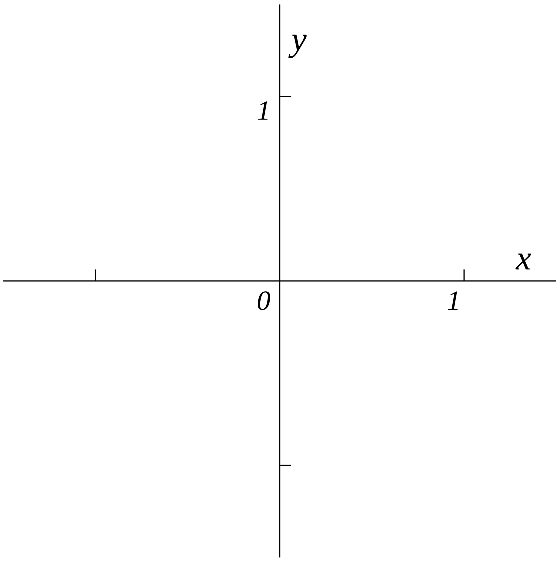
$$\int_0^1 \left(1+x^3\right)^{5/2}\,dx \approx 1.92$$

Here you will use the Trapezoid Rule to get this approximation.

a) First, find n (the number of subdivisions) so that you will be sure that the Trapezoidal Rule estimate with n subdivisions will be within 10^{-2} of the true value of the definite integral. (You may use the error bound $\dfrac{K(b-a)^3}{12n^2}$ where K is an overestimate of the magnitude of the second derivative.)

b) Write down the sum resulting from applying the Trapezoid Rule which you assert is within $\pm.01$ of the integral's value. Then use your calculator to actually compute this Trapezoid Rule approximation to **5** decimal places and report this result.

(20) 5. a) Sketch the polar curve $r = \cos(2\theta)$ and the circle $r = \frac{1}{2}$.



b) Find the area of that part of the plane which is both inside the curve and outside the circle.

Note: you probably want to take as much advantage of symmetry as possible.

(20) 6. I've been told by a silicon friend that

$$\int_0^\infty \frac{1}{(x+1)(x^2+1)}\, dx = \frac{1}{4}\pi$$

Check that this is correct using calculus.

Comment: you should first find an antiderivative of the integrand, and then carefully evaluate the improper integral.

(24) 7. True or false? If false, give an example to show that the implication is not true. If true, briefly explain why.

a) If $\{a_k\}$ is a **sequence** of real numbers so that $\{|a_k|\}$ converges, then $\{a_k\}$ must converge.

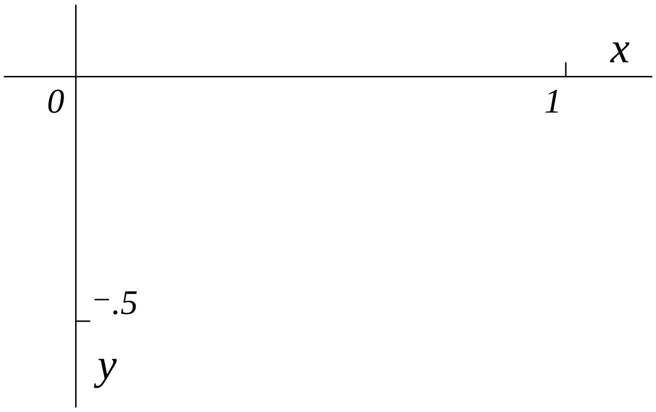
b) If $\{a_k\}$ and $\{b_k\}$ are **sequences** of real numbers and L is a real number so that $\lim_{k\to\infty} a_k = L$ and $\lim_{k\to\infty} b_k = L$, then $\lim_{k\to\infty} \frac{a_k}{b_k} = 1$.

c) If $\{a_k\}$ is a **sequence** of real numbers and L is a real number so that $\lim_{k\to\infty} a_k = L$, then $\lim_{k\to\infty} (a_{k+1} - a_k) = 0$.

d) If $\{a_k\}$ is a **sequence** of real numbers so that $\lim_{k\to\infty} (a_{k+1} - a_k) = 0$, then $\{a_k\}$ must converge.

(20) 8. Consider the function $L(x) = x \ln x$.

a) Sketch a graph of this function on the interval of x's between 0 and 1.



b) Since we know that $\lim_{x \to 0^+} \ln x = -\infty$, the visible behavior of $L(x)$ as $x \to 0^+$ is interesting. Explain this behavior with a limit computation.

c) Confirm with calculus that $\int_0^1 L(x)\, dx = -\frac{1}{4}$. (Be careful since the lower limit makes this officially an improper integral.)

(20) 9. Suppose that

$$G(x) = \frac{e^{(5x^2)}}{1-x}$$

a) Find the fourth degree Taylor polynomial, $T_4(x)$, for $G(x)$.

Comment: please *don't* compute the Taylor polynomial directly. Instead, begin with known results about the exponential function and the geometric series, and use algebra.

b) Suppose that $T_{\mathbf{3}}(x)$ is used instead of $G(x)$ when x is a very small non-zero number. Use information obtained in part a) to explain if you would expect that $T_3(x)$ would be greater than $G(x)$ or less than $G(x)$. Briefly explain your answer.

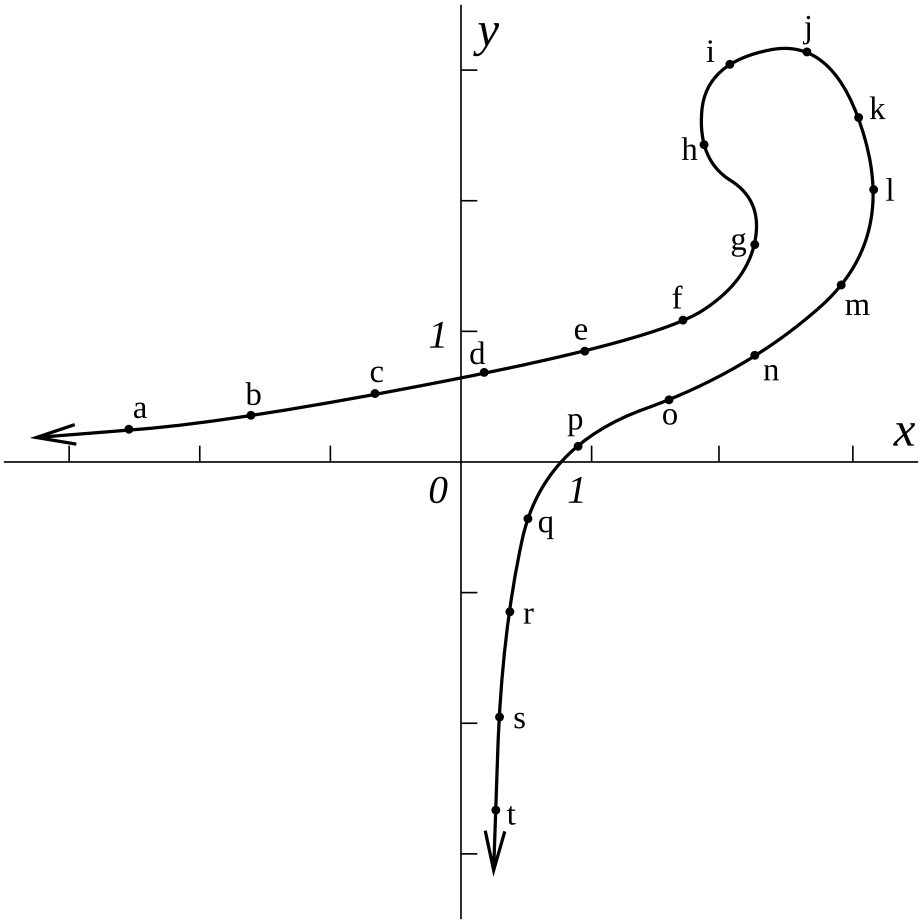
Comment: no computation beyond what has been done already for part a) is needed for this problem.

c) Use information obtained in part a) to evaluate $G^{(4)}(0)$.

Comment: you don't need to "simplify" any answer you give, and your answer should be easy to write using what you've done in a).

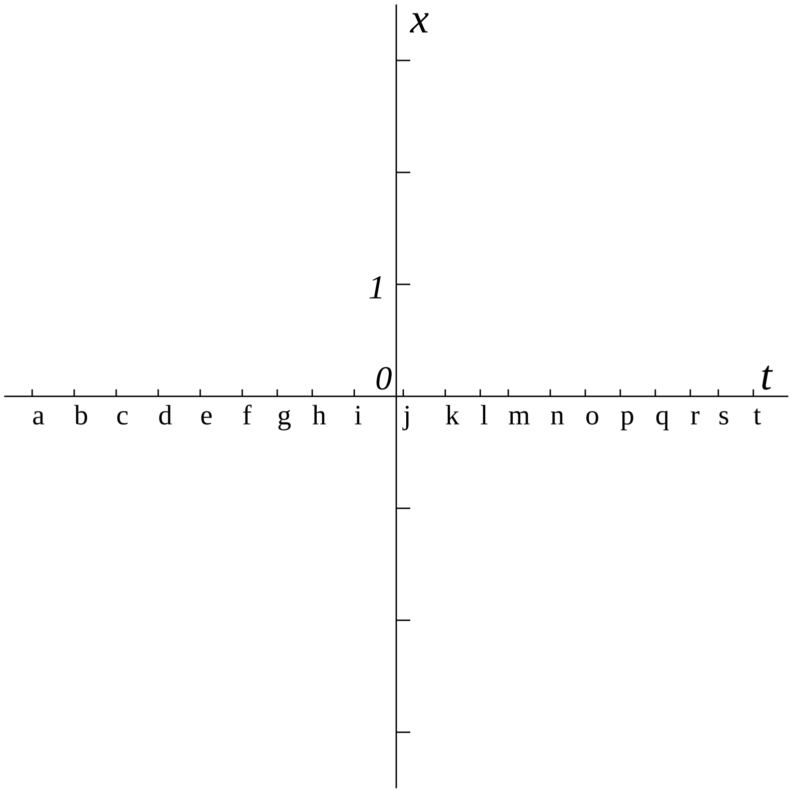
(20) 10. Below is the graph of a parametric curve. The alphabetic letters close to it are, in order, the increasing values of the parameter, t, which describes the curve using two functions, $x = \mathcal{F}(t)$ and $y = \mathcal{G}(t)$.
On the next page, sketch graphs of the functions $\mathcal{F}$ and $\mathcal{G}$ as well as you can. Note any intercepts of the coordinate axes of these graphs, and any local maxima and minima, and any asymptotic properties of these curves. You should write brief explanatory statements next to any interesting features of the graphs, such as maxima or minima or asymptotic behavior.

You may wish to detach this page here to make it easier to complete the graphs on the next page.

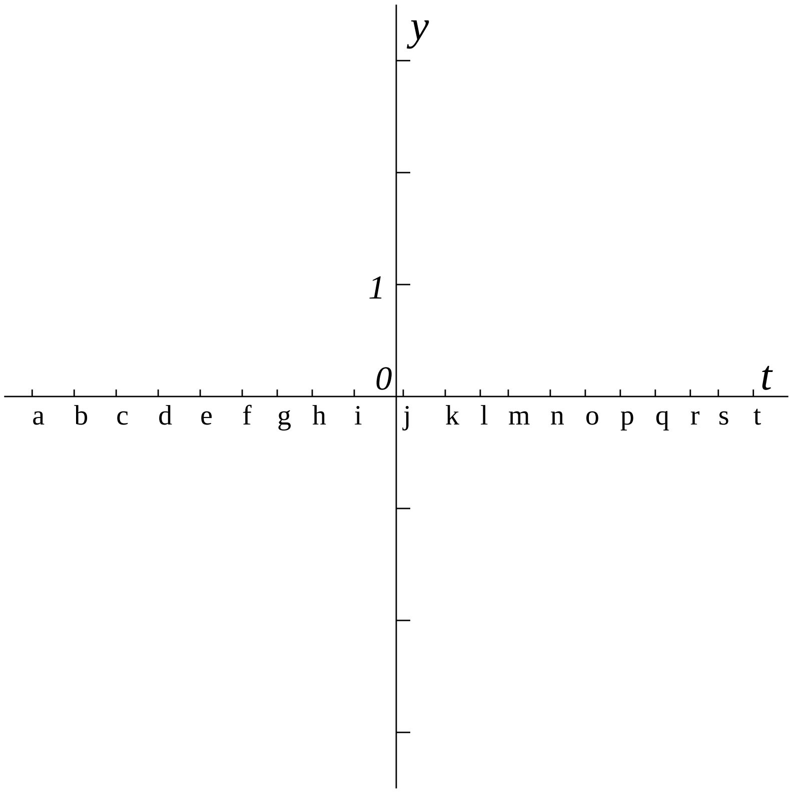


Remember to indicate as well as you can on each graph:

- Any intercepts of coordinate axes
- Any local maxima or minima
- Any asymptotic behavior



Your graph of $x = \mathcal{F}(t)$



Your graph of $y = \mathcal{G}(t)$

FINAL EXAM for MATH 192:03

December 18, 1996

NAME (*please print*): ______________________________

SIGNATURE: ______________________________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use TWO sheets of notes and any standard calculator without a QWERTY keypad on this exam. You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	16	
2	20	
3	20	
4	20	
5	20	
6	20	
7	24	
8	20	
9	20	
10	20	
Total Points Earned:		