Yet more calculus problems for 640:192:03

1. a) For x near 0, $\sin x$ is well-approximated by its tangent line at $x = 0$. What is this tangent line?

b) For many purposes, approximation over an interval is preferred over approximation near a point. There are many criteria for assessing the accuracy of such approximations. One is *mean-square error*. The mean-square error between a straight line $y = Ax$ going through the origin and the function $\sin x$ over the interval $[0, 1]$ is given by the definite integral

$$\int_0^1 (\sin x - Ax)^2 \, dx$$

Find the A which minimizes this integral. (Hint: expand the integrand and compute everything. You'll get a simple function of A. Minimize this function.)

c) Make a rough sketch over the unit interval $[0, 1]$ of $\sin x$ and the two straight lines found in a) and b).

2. Consider the function $G(x) = e^x \sin Nx$ on the interval $[0, 1]$.

a) With a sketch or otherwise, describe this function when $N = 3$ and when $N = 10$ and when $N = 10^{10}$.

b) Compute $\displaystyle\int_0^1 G(x)\, dx$ if $N = 10^{10}$.

c) Explain b)'s result in relation to a).

3. The following information is known about a function T and its derivatives:

x	$T(x)$	$T'(x)$	$T''(x)$
1	2	-2	2
2	3	6	5
3	7	4	-4
4	2	5	7

a) Compute $\displaystyle\int_2^3 T'(x)\, dx$.

b) Compute $\displaystyle\int_2^3 T''(x)\, dx$.

c) Compute $\displaystyle\int_2^3 x dx$.

d) Compute $\displaystyle\int_2^3 xT''(x)\, dx$.

Better not look at b) and c)! Integrate by parts.

e) Compute $\displaystyle\int_2^3 x^2 T'''(x)\, dx$.

And again and again.