Yet even more calculus problems for 640:192:03

1. a) Compute $\int_1^2 \frac{1}{x^2}\,dx$.

b) Suppose α is a small positive number. Compute $I_\alpha = \int_1^2 \frac{1}{x^2+\alpha}\,dx$. What is $\lim_{\alpha\to 0^+} I_\alpha$? (You *must* compute this from your final formula for I_α, not just plug $\alpha = 0$ into the integral. To see why, do problem 3.)

c) Suppose β is a small positive number. Compute $J_\beta = \int_1^2 \frac{1}{x^2-\beta}\,dx$. What is $\lim_{\beta\to 0^+} J_\beta$?

d) Draw some typical pictures of the areas computed by I_α and J_β (for small α and β) and the area computed in a). Do these help to explain any coincidences?

e) Is the function $F(t) = \int_1^2 \frac{1}{x^2+t}\,dx$ (defined for $t > -1$) continuous at $t = 0$? Use the definition of continuity (see Stewart, p.80) and your answers for a), b), and c).

2. Many integrals can be done with *rationalizing substitutions* which change the integrals into integrals involving rational functions. In turn, these integrals can be computed (theoretically) using partial fractions. Here are some examples of such integrals; please compute them.

a) $\int \frac{1}{(\sqrt{x}+1)(\sqrt{x}+2)}\,dx$

b) $\int \frac{e^x+1}{e^{2x}+1}\,dx$

c) $\int \frac{dx}{\sqrt{1+\sqrt{1+x}}}$

d) $\int \frac{\cos\theta}{1-(\sin\theta)^2}\,d\theta$ *

3. Suppose $F_A(x) = A^2(1-x)x^A$ (here A is a positive real number).

a) Sketch $F_1(x)$, $F_{10}(x)$, and $F_{10^{10}}(x)$ (for $0 \le x \le 1$).

b) Suppose $g(A) = \int_0^1 F_A(x)\,dx$. Compute $g(A)$. What is $\lim_{A\to\infty} g(A)$?

c) Again, consider $0 \le x \le 1$ only. Suppose $H(x) = \lim_{A\to\infty} F_A(x)$. Compute $H(x)$ and then $\int_0^1 H(x)\,dx$.

d) Parts b) and c) compare $\lim_{A\to\infty} \int_0^1 F_A(x)\,dx$ and $\int_0^1 \lim_{A\to\infty} F_A(x)\,dx$. Do the pictures in part a) help you to understand the results?

* You've just integrated sec. I hope you got ln(sec + tan)!