Calculus problems for 640:192:03

Do two of these three problems, please.

1. a) Graph $\ln x$ and x on the interval $[2,10]$.

b) Prove with calculus that $x > \ln x$ on the interval $[2,\infty)$.

c) Show that $\displaystyle\int_2^\infty \frac{1}{\ln x}\,dx$ diverges.

d) Does $\displaystyle\int_2^\infty \frac{1}{(\ln x)^2}\,dx$ converge? Why?

Comment Analysis of the question in d) does *not* follow from what is done in a), b) and c). There is a similar approach which I would like you to discover. Hints can/will be supplied!

2. Test each of the following integrals for convergence or divergence, showing the method used. Note: there is more than one thing to worry about in each of these integrals, and none of them can be evaluated in any simple form.

a) $\displaystyle\int_0^2 \frac{e^{\cos x}}{\sqrt{x^3(2-x)}}\,dx$

b) $\displaystyle\int_0^2 \frac{e^{\cos x}}{\sqrt{x(2-x)}}\,dx$

c) $\displaystyle\int_2^\infty \frac{e^{\cos x}}{\sqrt{x(x-2)}}\,dx$

d) $\displaystyle\int_2^\infty \frac{e^{\cos x}}{\sqrt{x^3(x-2)}}\,dx$

3. The integral $\displaystyle\int_1^\infty \frac{e^{-x^3}}{1+x^2}\,dx$ certainly converges. One simple way to try to approximate its value is to write it as the sum of two integrals:

$$\int_1^\infty \frac{e^{-x^3}}{1+x^2}\,dx = \int_1^T \frac{e^{-x^3}}{1+x^2}\,dx + \int_T^\infty \frac{e^{-x^3}}{1+x^2}\,dx$$

where T is some large number. Then any of our numerical techniques could be used to approximate the first (proper!) definite integral on the right-hand side of the equation. We could just *neglect* the second integral if we knew it was small.

a) Compare the second integral to an integral over the same interval of a simpler function to try to find a T so that the improper integral to be dropped will have a value smaller than 10^{-5}. *There is no unique correct answer to this part of the problem!*

b) Use a numerical technique to try to evaluate the first integral with an error of magnitude at most 10^{-5}. *You will need a programmable calculator or a computer for this part!*

c) Compute $\displaystyle\int_1^\infty \frac{e^{-x^3}}{1+x^2}\,dx$ with an error of magnitude at most $2\cdot 10^{-5}$.