640:192:03 # Calculus problems for 640:192:03 **11/4/96**

Power series are definitely *not* the only kinds of series that are considered in practical applications. One type of series used to investigate periodic phenomena is **Fourier Series**. Here is a problem about one such series whose solution I would like you to turn in.

Consider the function defined by

$$F(x) = \sum_{n=1}^{\infty} \frac{1}{2^n} \sin(nx)$$

a) Explain why the series defining $F(x)$ converges absolutely for all values of x, and then conclude that it must converge for all x's. Can you give *some* estimate for how big or how small $F(x)$ could be? Could $F(x) = 53$? Could $F(x) = -6$?

b) Compare $F(x)$ and $F(x + 2\pi)$. Explain your assertion.

c) Suppose the function $F_N(x)$ is defined by

$$F_N(x) = \sum_{n=1}^{N} \frac{1}{2^n} \sin(nx)$$

(so $F_N(x)$ is a partial sum of the series for $F(x)$). Can you find a specific value of N so that $F_N(x)$ and $F(x)$ are always within .01 of each other?

d) Use the value of N you found in c) and draw a graph of $F_N(x)$ on the interval $[0, 2\pi]$. What can you conclude about the appearance of the graph of $F(x)$?

A festival of series:

Do these series converge? Do they converge absolutely? Do they converge conditionally? If there's an x in the series, for which values of x do the series converge or converge absolutely or conditionally? We will discuss these questions in class.

$$\sum_{n=1}^{\infty} \frac{1}{n^2+\sqrt{n}}$$

$$\sum_{n=2}^{\infty} \frac{1}{n^2-n}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{(3n)!}{(n!)^3} x^n$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{(3n)!}{n!(2n)!} x^n$$

$$\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$$

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4+1}}$$

$$\sum_{n=1}^{\infty} 7n^5 x^n$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{(n+4)^2}$$

$$\sum_{n=1}^{\infty} \frac{(n^4+1)^2}{(n^2+1)^{3/2}}$$

$$\sum_{n=2}^{\infty} \frac{(-1)^n n}{n+\ln n}$$

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$$

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n^n}$$

$$\sum_{n=1}^{\infty} \frac{\sin(203n^5-47n^2+29.888)}{n^{1.008}}$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n \ln n}$$