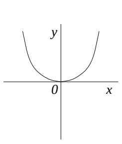
640:251:19-20 **Answers to quiz given yesterday** **9/27/96**

Problem #1 Find parametric equations for a line tangent to the curve $\begin{cases} x = t^2 \\ y = t^2 - t \\ z = 1 - t^3 \end{cases}$ when $t = 1$. (I may have written $y = t^2 - 1$ in one section – change the answer suitably.) Your parameter for the line must be w.

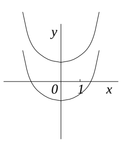
Answer $\overrightarrow{r}(t) = \langle t^2, t^2 - t, 1 - t^3 \rangle$ so $\overrightarrow{v}(t) = \langle 2t, 2t - 1, -3t^2 \rangle$ and when $t = 1$ we get a point the line goes through and a vector in the direction of the line, so equations for the line are: $\begin{cases} x = 2w + 1 \\ y = 1w + 0 \\ z = -3w + 0 \end{cases}$.

Problem #2 Sketch three level curves associated with the function $f(x, y) = (y - x^2)^2$. You must have curves which aren't empty and you must tell me what the constants are.

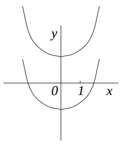
Answer $f(x, y)$ is a square, so its values are non-negative. All of the level curves associated with values < 0 must be empty. When is $f(x, y) = 0$? Well, $(y - x^2)^2 = 0$ implies $y - x^2 = 0$ implies $y = x^2$. So here's that level curve (below). When $f(x, y) = 1$ something a little different happens. If $(y - x^2)^2 = 1$ then $y - x^2 = \pm 1$ so $y = x^2 \pm 1$ and we get *two* parabolas! Similarly, if $(y - x^2)^2 = 2$ then $y = x^2 \pm \sqrt{2}$.



$f(x, y) = 0$



$f(x, y) = 1$



$f(x, y) = 2$