(17) 1. The two planes

$$x + 2y - 3z = 5$$

and

$$3x - y + z = 2$$

intersect in a straight line. Find parametric equations of this line.

(12) 2. Find an equation for the plane normal to (that is, perpendicular to) the curve

$$\begin{cases} x = t^2 \\ y = 1 - t^3 \\ z = t \end{cases}$$

at the point where $t = 1$.

(18) 3. Suppose K is a constant and

$$F(x, y) = xe^{Kx^2y}$$

a) Compute $\dfrac{\partial F}{\partial x}$ and $\dfrac{\partial^2 F}{\partial x^2}$ and $\dfrac{\partial F}{\partial y}$.

b) There is one value of K so that $F(x, y)$ satisfies the equation

$$\frac{\partial^2 F}{\partial x^2} = 2y^2 \frac{\partial F}{\partial y} + 3yF$$

for all values of x and y. Find that value of K and verify that $F(x, y)$ then does satisfy the equation.

(18) 4. Suppose $Q(x, y)$ is a differentiable function of two variables and that

$$A = Q(1,1) \qquad B = \frac{\partial Q}{\partial x}(1,1) \qquad C = \frac{\partial Q}{\partial y}(1,1)$$

$$D = \frac{\partial^2 Q}{\partial x^2}(1,1) \quad E = \frac{\partial^2 Q}{\partial x \partial y}(1,1) \quad F = \frac{\partial^2 Q}{\partial y^2}(1,1)$$

Suppose also that $g(t) = Q(t^2, t^3)$.

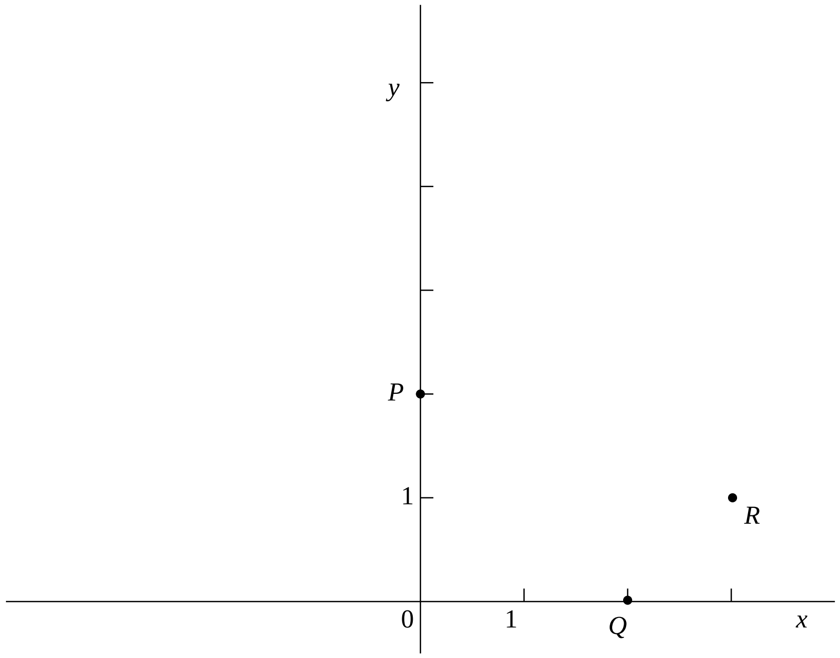
a) Compute $g'(t)$ using the chain rule. Then verify that $g'(1) = 2B + 3C$.

b) Use the first part of your answer to a) and the chain rule to compute $g''(t)$. Then verify that $g''(1) = 2B + 6C + 4D + 12E + 9F$.

(15) 5. Sketch the three level curves of the function

$$W(x,y) = \frac{y}{x^2+1}$$

which pass through the points $P = (0,2)$ and $Q = (2,0)$ and $R = (3,1)$. Be sure to label each curve with the appropriate function value and be sure that your drawing is clear and unambiguous.



(20) 6. Suppose the curve C has position vector

$$\overrightarrow{r}(t) = \cos t\,\overrightarrow{i} + \sin t\,\overrightarrow{j} + t^2\,\overrightarrow{k}$$

a) What is the unit tangent vector to C when $t = \pi$?

b) Write an integral for the length of the part of C between $t = 0$ and $t = 2\pi$.
DO NOT EVALUATE THE INTEGRAL.

c) The curvature of this curve is given by the formula

$$\kappa(t) = \sqrt{\frac{5+4t^2}{(4t^2+1)^3}}$$

which you should **not** verify. Assume it is correct. Even though the first two components of this curve describe uniform circular motion, $\lim_{t\to\infty} \kappa(t) = 0$. Explain briefly why this can happen, using complete English sentences possibly assisted by properly labelled diagrams.

EXAM 1 for MATH 251, sections 19 and 20

October 11, 1996

NAME (*please print*): ____________________

SIGNATURE: ____________________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use both sides of one sheet of notes and any standard calculator without a QWERTY keypad on this exam.

You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	17	
2	12	
3	18	
4	18	
5	15	
6	20	
Total Points Earned:		