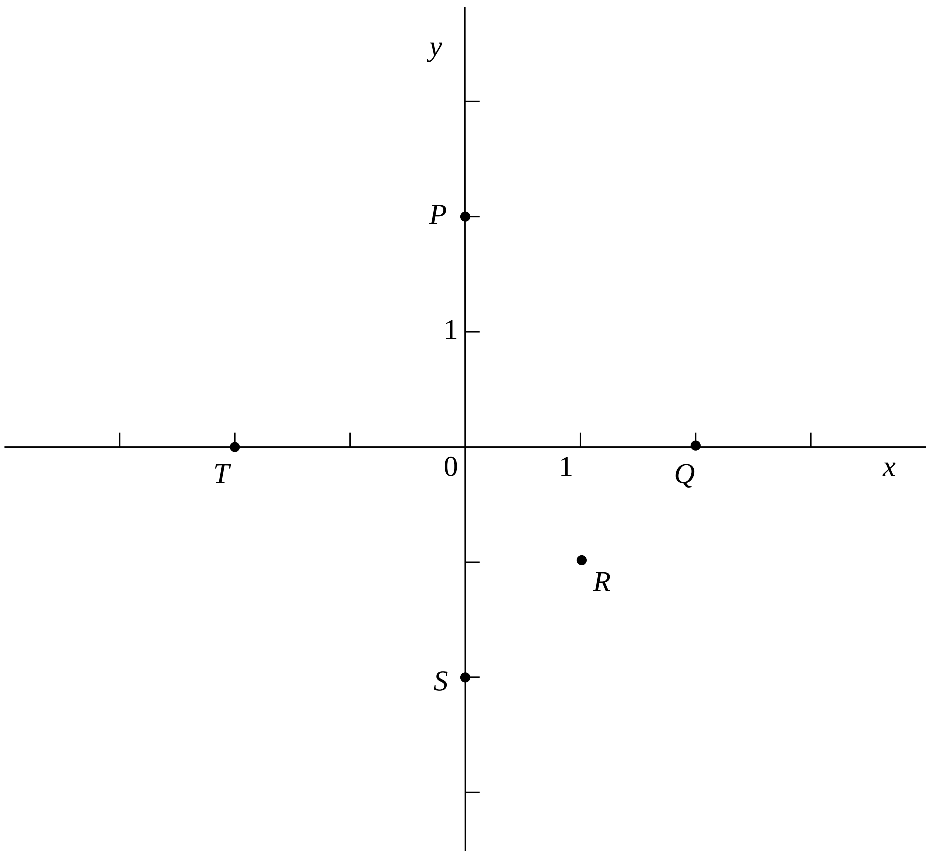
(15) 1. For what values of t are $\mathbf{u} = t\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{v} = t\mathbf{i} + t\mathbf{j} - 2\mathbf{k}$ perpendicular? Are there values of t for which $\mathbf{u}$ and $\mathbf{v}$ are parallel?

(20) 2. Sketch the three level curves of the function

$$W(x,y) = ye^x$$

which pass through the points $P = (0,2)$ and $Q = (2,0)$ and $R = (1,-1)$. Be sure to label each curve with the appropriate function value and be sure that your drawing is clear and unambiguous.

Also, sketch on the same axes the vectors of the gradient vector field ∇W at the points P and Q and R and S and T. The point $S = (0,-2)$ and the point $T = (-2,0)$.



(25) 3. Suppose that $F(x, y)$ is a function all of whose second partial derivatives exist and are continuous. Suppose you also know that

$$F(0,0) = a,\ \frac{\partial F}{\partial x}(0,0) = b,\ \frac{\partial F}{\partial y}(0,0) = c,\ \frac{\partial^2 F}{\partial x^2}(0,0) = d,\ \frac{\partial^2 F}{\partial x \partial y}(0,0) = e,\ \frac{\partial^2 F}{\partial y^2}(0,0) = f$$

and that

$$G(s,t) = F(3s + 2t, st)$$

Compute

$$G(0,0)\ ,\ \frac{\partial G}{\partial s}(0,0)\ ,\ \frac{\partial G}{\partial t}(0,0)\ ,\ \frac{\partial^2 G}{\partial s^2}(0,0)\ ,\ \frac{\partial^2 G}{\partial s \partial t}(0,0)\ ,\ \text{and}\ \frac{\partial^2 G}{\partial t^2}(0,0)$$

in terms of a, b, c, d, e, and f. (There are **six** quantities to be computed, and you must show your work!)

(20) 4. Find all critical points of each function. Describe (as well as you can) the type of each critical point. Explain your conclusions.

a) $f(x, y) = x^3 + y^3 + 6xy$

b) $g(x, y, z) = x^{200} + y^{400} + z^{600}$

(20) 5. a) Find the equation of the plane tangent to the surface $x^2yz = 6$ at the point $(1, 2, 3)$.

b) A particle has position vector given by $\mathbf{r}(t) = \cos(2t)\mathbf{i} + \sin(t^2)\mathbf{j} - 3t\mathbf{k}$. Find parametric equations for a line tangent to the path of this particle when $t = 0$.

c) The plane found in a) and the line found in b) intersect. Find the point of intersection.

(20) 6. Compute the integral of the function $e^{-\left(x^2+y^2\right)}$ over the region in the first octant which is under the paraboloid $z = 1 - \left(x^2 + y^2\right)$. (The *first octant* in (x, y, z)–space is the collection of points which have $x \geq 0$, $y \geq 0$, and $z \geq 0$.)

Note: be careful! π and e both *definitely* appear in the answer!

(20) 7. Consider the iterated integral

$$\int_0^2 \int_{y^4}^{y^2+12} y \, dx \, dy$$

a) Compute this integral

b) Sketch the region in the $(x, y)-$plane over which this integral is computed.

c) Write an expression in $dy\ dx$ order (an integral or sum of integrals) which is the same as the original iterated integral in this problem. You need **not** compute this expression.

(20) 8. Suppose a vector field is defined by $\mathbf{F}(x, y, z) = \left(y^2 z\right)\mathbf{i} + (2xyz)\,\mathbf{j} + \left(xy^2 + 4z\right)\mathbf{k}$.

a) Determine whether there is a scalar function $P(x, y, z)$ defined everywhere in space such that $\nabla P = \mathbf{F}$. If there is such a P, find it; if there is not, explain why not.

b) Compute the integral $\displaystyle\int_W \mathbf{F} \cdot \mathbf{T}\, ds$, where W is the circular helix whose position vector is given by $\mathbf{R}(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j} + t\,\mathbf{k}$ for $0 \le t \le 2\pi$. Use information gotten from your answer to a) to help if you wish.

(15) 9. Evaluate the line integral

$$\int_P \left(2xy^2 + \tan x\right) dx + \left(-3x^2 - e^y\right) dy$$

where P is the rectangle with vertices $(0,1)$, $(1,1)$, $(1,3)$, and $(0,3)$ oriented in the usual (counterclockwise) fashion.

Note: use *any* method you want to evaluate this integral. Some methods are easier than others.

(25) 10. Find the total flux upward through the upper hemisphere (where $z \geq 0$) of the sphere

$$x^2 + y^2 + z^2 = a^2$$

of the vector field

$$\mathbf{T}(x, y, z) = \left(\frac{x^3}{3}\right)\mathbf{i} + \left(yz^2 + e^{\sqrt{zx}}\right)\mathbf{j} + \left(zy^2 + y + 2 + \sin\left(x^3\right)\right)\mathbf{k}$$

Note: do *not* attempt to compute this directly! Use the Divergence Theorem on some "simple" solid to change the desired computation to the computation of a triple integral and a much simpler flux integral. Evaluate those integrals, taking as much advantage of symmetry as possible.

FINAL EXAM for MATH 251, sections 19 and 20

December 17, 1996

NAME (*please print*): ______________________________

SIGNATURE: ______________________________

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use one sheet of notes and any standard calculator without a QWERTY keypad on this exam. You may use no other materials.

Problem Number	Possible Points	Points Earned:
1	15	
2	20	
3	25	
4	20	
5	20	
6	20	
7	20	
8	20	
9	15	
10	25	
Total Points Earned:		