Mathematics 251: Lab 2 QUADRIC SURFACES

A quadric surface is the graph of a second-degree equation in three variables x, y, and z. The most general form of such an equation is:

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0,$$

where $A, B, C, D, E, F, G, H, I$, and J are constants.

In this lab, we use Maple to help visualize some types of quadric surfaces that can arise from equations of the above form.

Please turn in only the printout of your Maple worksheet. Include explicit answers to all questions asked, using the **text** feature of Maple to insert them in the worksheet (don't write them by hand). Insert graphs directly into your worksheet; instructions were given in the initial handout. Remove from the worksheet any extraneous material such as the output of commands like `with(plots);` and any errors you have made.

1. An ellipsoid is a quadric surface of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

where a, b, and c are positive constants.

Note that if we solve for z, we get the two surfaces

$$z = c\sqrt{1 - x^2/a^2 - y^2/b^2}, \qquad z = -c\sqrt{1 - x^2/a^2 - y^2/b^2}.$$

1a. Use the `plot3d` command to plot the upper half of the ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} = 1$. One way to do this is to first define constants a, b, and c by typing

```
a:=2; b:=3; c:=5;
```

To plot the upper half of the ellipsoid, then type

```
plot3d(c*sqrt(1 - x^2/a^2 - y^2/b^2), x= -3..3, y=-3..3;
```

Also obtain a plot in which you include the option `axes=BOXED` by typing

```
plot3d(c*sqrt(1 - x^2/a^2 - y^2/b^2), x= -3..3, y=-3..3, axes=BOXED);
```

Note that if you forget to include this option in the `plot3d` command, you can also obtain the option after Maple draws the plot by clicking the mouse on **Axes**, then dragging the mouse down, releasing on **Boxed**. Clicking the middle mouse button will then cause Maple to redraw the plot using the Boxed option.

1b. Obtain on a single plot both halves of the ellipsoid of part (a) by typing

```
plot3d({c*sqrt(1 - x^2/a^2 - y^2/b^2),-c*sqrt(1 - x^2/a^2 - y^2/b^2)},
x= -3..3, y=-3..3, axes=BOXED);
```

Note the use of the braces {} when plotting more than one function. Also note that Maple has some problem plotting points where the surface is almost parallel to the z axis.

1c. Use Maple to help determine the curve formed by the intersection of the ellipsoid of part (a) with the plane $z = 4$ by typing

```
plot3d({c*sqrt(1 - x^2/a^2 - y^2/b^2),-c*sqrt(1 - x^2/a^2 - y^2/b^2),
4}, x= -3..3, y=-3..3, axes=BOXED);
```

Use the **text** feature of Maple to state explicitly what type of curve you have found. Also determine without the use of Maple the equation of the curve of intersection and enter it into your Maple worksheet, making sure to label your equation (e.g., Equation of curve of intersection).

1d. Obtain a plot which clearly shows the curve formed by the intersection of the ellipsoid of part (a) with the plane $x = 1$. Note that since Maple cannot draw planes parallel to the vertical axis, you will have to modify the approach used in part (c).

2. The surface

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

is called an elliptic paraboloid and the surface

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

is called an hyperbolic paraboloid.

2a. Plot the elliptic paraboloid corresponding to $a = 2$, $b = 3$, $c = 5$.

2b. Obtain a plot which shows the intersection of this paraboloid with the plane $z = 4$. Clearly state what type of curve is formed by this intersection.

2c. Plot the hyperbolic paraboloid corresponding to $a = 2$, $b = 3$, $c = 5$.

2d. Obtain a plot which shows the intersection of this paraboloid with the plane $z = 4$. Clearly state what type of curve is formed by this intersection.

3. Consider the family of surfaces given by

$$z = x^2 + y^2 + dxy$$

for different values of the constant d. Although these surfaces are not exactly of the form described in the previous problem, for some values of the constant d they are elliptic paraboloids and for others they are hyperbolic paraboloids. In this problem we try to determine a transitional value of d for which the surface changes from one type to another by viewing the graphs of this surface for various values of the constant d.

Plot surfaces corresponding to some integer values of d satisfying $0 \leq d \leq 9$ over the range $-1 \leq x \leq 1$, $-1 \leq y \leq 1$.

2a. From among these surfaces, choose one which is an elliptic paraboloid, and place it in your Maple worksheet, labeling it as elliptic and giving the value of d to which it corresponds.

2b. From among the above surfaces, choose one which is a hyperbolic paraboloid, and place it in your Maple worksheet, labeling it as hyperbolic and giving the value of d to which it corresponds.

2c. From among the above surfaces, choose one which is a transitional surface, and place it in your Maple worksheet labeling it as transitional and giving the value of d to which it corresponds.

In order to determine the type of paraboloid, look for features similar to features in one of the plots of the previous problem. To distinguish these features, it may be helpful to look at a surface from different viewpoints. This is accomplished by first moving the mouse pointer to the graphics window displaying the surface. Then click and hold down the left mouse button and move the mouse. An outline of a cube will appear which rotates as you move the mouse. To display the surface from the same viewpoint as the cube (determined by the angles Phi and Theta), release the left mouse button and click once on the middle mouse button. For each of the plots requested above, try several different viewpoints (e.g., Theta=45, Phi=90 and Theta = -45, Phi =85). Place only the best pictures (more than one may be needed in some cases to clearly show the type of surface) for each value of d you have chosen in your Maple worksheet.

4a. Use the `plot3d` command to plot the surfaces

$$z = x^2 + y^2, \qquad z = 1 - y^2$$

on a common screen using the domain $|x| \leq 1.2$, $|y| \leq 1.2$.

4b. It is often useful to have a plot of the projection of the curve of intersection of two surfaces onto one of the coordinate planes. This can be easily done using the `implicitplot` command. In order to activate this command, you first need to type `with(plots):`.

Obtain a plot of the projection on the xy plane of the intersection of the surfaces of part (a) by typing

```
with(plots):
implicitplot(x^2+y^2 = 1-y^2,x=-1.2..1.2,y=-1.2..1.2);
```

4c. To see the real shape of the curve, obtain a plot adding the option `scaling=CONSTRAINED` to your command, i.e., type

```
implicitplot(x^2+y^2 = 1-y^2,x=-1.2..1.2,y=-1.2..1.2,scaling=CONSTRAINED);
```

4d. Without using Maple, determine the equation of the curve you have plotted in part (c) and enter it into your Maple worksheet using the **text** feature.