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To: 251 Course Coordinator, Undergraduate Vice-chair & other interested parties

From: Steve Greenfield

Re: More about changing calculus, especially Math 251

Here are more comments about Math 251, and, inferentially, about all our efforts in changing calculus. There's information which may be interesting or amusing.

• **The good old boys in the good old days**

I had a year of calculus in high school using the second edition of Thomas's book as text. In college thirty years ago I took third semester calculus (calculus of several variables) at Columbia. The course was given in lecture/recitation format. The recitations were taught by a math grad student and discussed textbook problems. The lecturer was the late topologist, P. A. Smith, who spoke from the stage of a large auditorium, most of the time wearing suspenders over a long-sleeved white shirt and tie. I still remember him drawing a large arrow on the board and declaring, "Gentlemen*, *this* is a vector!"

The students were engineering, math, physics, and chemistry majors. The text was by Sherwood and Taylor (third edition), probably chosen because it was *not* by Thomas, whose book everyone else used. There is **no** vector calculus in either book. There's one thin section on line integrals in the Thomas text. But there's *nothing* about **surface integrals** or about the **Green's-Gauss's-Stokes-Divergence Theorems**. About 25% of Math 251 is devoted to these topics. What *did* we do in that course years ago, and when did we learn vector calculus?

I learned some vector calculus in the Electricity and Magnetism course given by the Physics Department. I saw an abstract version of Stokes Theorem late in the second semester of an advanced calculus course and didn't understand it for years until I saw applications to differential geometry and partial differential equations.

A colleague who attended MIT a few years later reported that he, too, didn't learn Green-Gauss-Stokes (G-G-S) as part of the ordinary calculus curriculum. There engineers were taught G-G-S in a special higher-level mathematical methods course and math majors learned a version in the second semester of an advanced calculus/analysis course.

Equipped with the miserly instincts of a packrat, I easily found my notes on several variable calculus taken long ago. The notes show a more careful presentation of what it means for a function of several variables to be differentiable than I do now, and there was more discussion of Jacobians than any of us do now. There are muddled notes about max/min in several variables motivated by Taylor's Theorem. I was confused, and dutifully memorized the second derivative test: notes weren't allowed on exams! I remember *not* understanding Lagrange multipliers at all (until, in fact, the second year of grad school). The exams I recall from Math 26 weren't theoretical. Each exam consisted of 6 to 8 computational problems. Long explanations weren't required, and few of the computations were elaborate or delicate.

* Columbia was then for men only. A few fierce women were in the classes, anyway.

Here's a comparison of the treatment of some topics in the early edition of Thomas and in our text by Stewart.

Topic	**old Thomas**		**current Stewart**	
	Pages	Problems	Pages	Problems
The chain rule in several variables	4	14	8	52
Beginning material on double integrals	4	10	10	54
Triple integrals in cyl. & spher. coords	2	7	4	38

The textbook "swell" (the increase in size, not necessarily in quality!) has been amazing. Do students need more help learning or do teachers need more help teaching now? Or do we have too many trees?

Conclusion We are trying to teach more hard topics to an assortment of "regular" students with a wider variety of majors at a state university than were taught in similar courses 20 or 30 years ago to students with technical majors at highly selective colleges. And that's without our pedagogical "improvements", which should make it easier to do more but which introduce their own costs.

- **New things for new students**

We've made major changes to Math 251 this semester which we hoped would all be improvements. Shrinking class size is surely good. But what about the others?

 # 1: The Text

I was one of the group who chose our **new textbook**. We hoped it had better explanations and was less "formulaic" than the text by Thomas & Finney. It moved towards "reformed calculus" without including the extended story-telling coupled with sociology which some of the calculus reform books seem to contain. Some aspects of the texts I used as a student and the two used most recently at Rutgers can be easily measured.

Authors	**Weight**	**Pages**	**Page size**
Thomas & Finney	5 lbs 5 oz	1215	$8.5'' \times 10.5''$
Stewart	4 lbs 12 oz	1114	$8.5'' \times 10.5''$
The old Thomas	2 lbs 6 oz	822	$6'' \times 9''$
Sherwood & Taylor	1 lb 13 oz	574	$6'' \times 9''$

Adequate engineers, scientists, and even mathematicians were educated using the old books. The current texts have many colored pictures and very wide margins. Stewart's text costs \$91.25 when new; the student answer book costs \$27.00. How do students and instructors use the text? How many of our students will save Stewart's book for 20 or 30 years, and should they? Students mainly use the text to find homework problems and to read examples. They rarely *read* it to learn from it. I believe that we should move towards a textbook–less calculus course, with a sort of formula sheet/example collection available. Textual assistance with alternative explanations could be referenced or given to the class as necessary. This could be done electronically, with the option of print-outs available.

We use the text almost solely to assign problems and rarely refer to other contents. In each section we assign 2 or 3 of the easiest problems, another 2 or 3 of the moderately difficult problems, and perhaps 1 or 2 of the more imaginative (tougher?) last third. I did fewer problems from Sherwood & Taylor. I was assigned a few problems per section and did more to prepare for exams. The department's suggested syllabus for Math 251 several years ago had **295** textbook homework problems. We now list about **265** such problems. I think most of my students several years ago worked hard enough. We reduced the workload by a few problems each week but now require other substantial tasks.

 # 2: Technology

Students with serious interests in scientific and technical subjects should know about programs such as `Maple` which nicely integrate symbolic, graphical and numerical aspects of mathematical computation. It's a good program and widely available at Rutgers. Math 251 students should learn about `Maple` and use it in this course and in others. My comments are mostly based on `Maple` use in my Math 251 class.

My students seem to have spent too much time learning how to use `Maple`. Some report stumbling at every missing punctuation mark. We may want to prepare a Math 251 package for `Maple`, adapting certain commands with syntax more understandable to our students. I'm also teaching a Math 192 class this semester, a much smaller group of students than any 251 class. I spent a period in a computer lab working on `Maple` with students. I emphasized that a user can't break the system or the program, that the program had many help facilities available, and that `Maple` "empowered" students to discover new ways of doing things. This is important if we want students to use `Maple`. Computer lab attendants usually know little about `Maple` and, even if they do, have little time to give individual help. It is *essential* that we schedule some organized instruction for every student early in the semester in a computer lab. I didn't and I was wrong.

We need to pay more attention to how much time the `Maple` labs take. Not every horror story is believable (especially from weaker students expressing complaints the night before an assignment's due) but I got enough negative comments from students I trust to conclude that we should prune some of the assignments and consider carefully what we want people to do. I cut down the last two assignments, and should have shortened the first. If the `Maple` assignments are too long, we increase the chance of receiving student work tainted by copying. Students should know the rewards for learning about `Maple` and not find it excessively difficult or time-consuming.

I startled students by urging them to try `Maple` on workshop assignments or even some of the more tedious textbook problems because I wanted them to use `Maple`. I hadn't adequately forseen some difficulties: some students live off campus, and some computer centers aren't open and functioning well some of the time. I have `Maple` at home and at work, and frequently carry a `Maple`-equipped laptop. Few students have such access.

 # 3: Workshops

Students in math courses should to be able to talk and write about mathematics. We use workshops to help achieve this goal. Workshops are classes where students work together in small groups on more unconventional "open-ended" problems. Discussion is encouraged, and written solutions are graded for both content and presentation.

Workshops have been quite useful in EXCEL which has small numbers of students in workshops three evenings a week with much attention given to written work. Workshops may not be as effective in Math 151 but are an improvement over what we used to have.

In Math 151, larger class size and less time in workshops decrease workshop usefulness. Also, the grading isn't as helpful (in EXCEL students are asked to redo unsatisfactory workshop writeups). The effectiveness drops somewhat more in Math 152, when students are less acquainted with the subject matter.

Almost all of the students entering Math 251 know little about several variable calculus. In 251 even routine manipulations seem sometimes to need the reinforcement of a standard recitation atmosphere: "Let's do problem 16. Now problem 18 ..." Students have real difficulties with such topics as setting up multiple integrals and dealing with the algebraic intricacies of Lagrange multipliers. They need more routine problem time, and I feel that increasing the recitation time from 55 to 80 minutes isn't enough if we want to devote a lot of that time to workshops. My students, at least, seem less interested in "playing the game" of forming discussion groups and pulling apart the problems. We push EXCEL students to do this with much reinforcement. Math 251 students have more competition for their time and interest, and are less willing to be manipulated towards these pedagogical ends. My sample of students may be biased, so I would like to know other instructors' experiences.

We probably need more time for the course if we want to have a real workshop component. Asking for more time may not be realistic in terms of the other demands for students' attention. If we want `Maple` in the course and some workshops, the number of recommended textbook problems should be reduced. Note that we gave over **350** textbook problems in Math 151. Do students really work on all these?

- **Experience refines changes**

My comments should not be read as praise to a vanished Golden Age. The situation now is more complex, because the students are different, with different preparation and ambitions. Some textbook changes are terrific – the pictures are great, and are not just indulging the MTV audience. Our technical tools are far superior to what was available years ago: drawing and looking at the appropriate pictures can now be done by everyone without expending huge effort and without an expensive text. As a student, I could find an antiderivative of $\arctan(x^2)$ by hand, and would have thought it a tedious but not ridiculous computation. My own teachers in their student days would probably have felt the same about computing a five decimal digit approximation to $\sqrt{235}$, an assignment I would have resented and found silly. Times change.

What are our objectives in this course? Do we want rote memorization, as was perhaps too much the case years ago? Do we want to teach simply use of a computer program?

What do our students want from the course? They may want an easy A, but not many are taking the course because of an idealistic hunger for ideas (that was never the case!). They must know how to think and learn about some mathematical topics, to connect these topics with other subjects, and to communicate their understanding effectively.

We should find ways to accomplish the joint goals of teaching and learning as well as we can within a "budget" that all partners in the enterprise can tolerate. For example, we shouldn't unreasonably burden the instructional staff with grading. Similarly we should not overload the students with assignments. We should make efforts to accommodate the varied learning styles of our students while still pursuing the intellectual goals we think are important: the beauty, precision, and power of the subject.