640:251:19-20 **9/12/96**

Problems, Week 2

I'll tell you one problem whose solution should be written up for next week's meeting. Your written solution should be of high-quality, with complete explanations given including any useful diagrams or equations. While I encourage you to discuss the problems with other students and with me, your written work must be your own explanation of the problem.

1. Suppose $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^3$, and the following is known:

$$5 \leq \|\mathbf{v}\| \leq 10, \quad 30 \leq \|\mathbf{w}\| \leq 40, \quad 30^\circ \leq \theta \leq 60^\circ$$

where θ is the angle between $\mathbf{v}$ and $\mathbf{w}$. Then find appropriate over– and under–estimates for the following real numbers using all of the information given:

$$\mathbf{v} \cdot \mathbf{w}, \quad \|\mathbf{v} \times \mathbf{w}\|, \quad \|\mathbf{v} + \mathbf{w}\|$$

2. a) What is the equation of a sphere with center $(1, 2, 3)$ which is tangent to the plane $3x + 2y + z = 5$?

b) Find the equation of a sphere of radius 5 which is tangent to both the planes $x-2y+2z = 3$ and $3x + 4z = 8$.

3. Let L be the line with vector equation $\mathbf{r} = \langle 1, 0, 3\rangle + t\langle -1, 2, 1\rangle$. Find equations for two orthogonal planes containing L, one of which passes through the origin. (Hint: find the equations one at a time).