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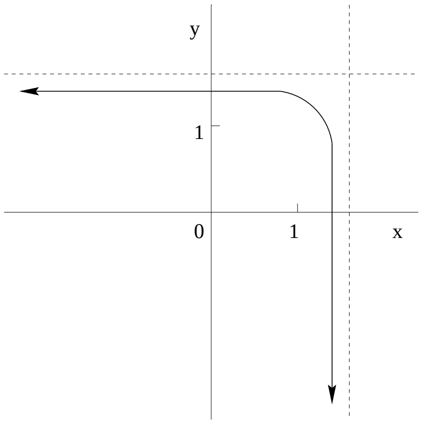
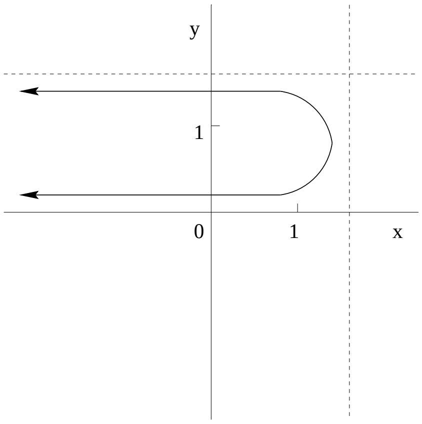
Problems, Week 4

9/26/96

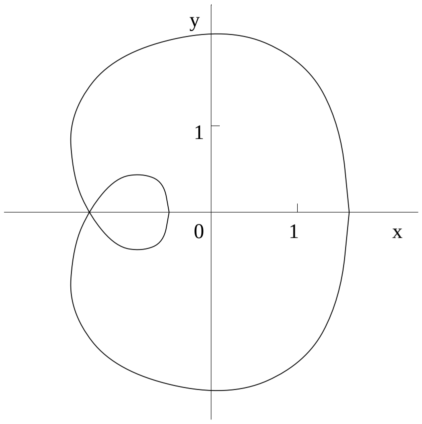
1. a) Suppose each of the following curves is parameterized "by arc length": that is, walk along them at unit speed. Make a reasonable sketch of the *curvature* of each curve as a function of the parameter. Discuss why your sketches are reasonable.



Curve I

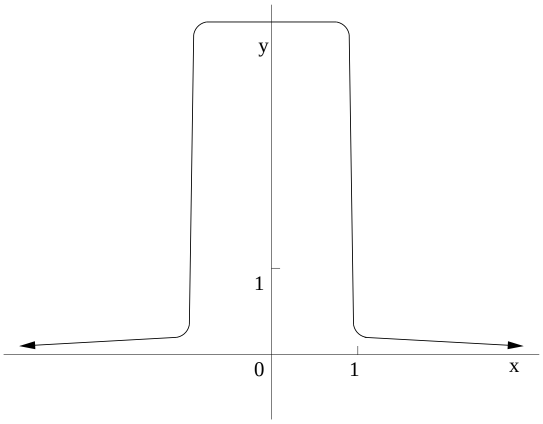


Curve II

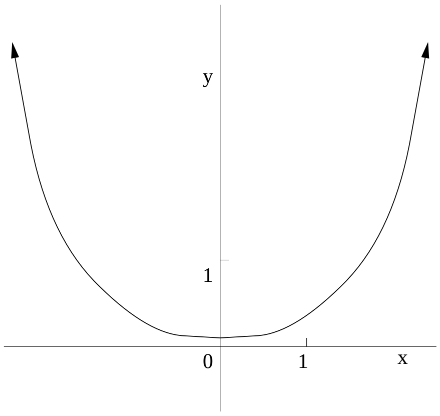


Curve III

b) The following is a sketch of the *curvature* of several curves. In each case, sketch a curve parameterized by arc length (so you travel along it at unit speed) whose curvature may reasonably have the graphs indicated. Discuss why your sketches are reasonable.



Curvature of curve A



Curvature of curve B

2. a) Sketch the surface

$$x^2 + y^2 + z^2 = 1.$$

To help with part b) below, think of this as sketching the equation $z^2 = 1 - (x^2 + y^2)$, and remember that squares must be non-negative. Where is the right-hand side zero, and what is z then? How many values of z are there when the right-hand side is positive?

b) Sketch the surface

$$z^2 = \left(1 - (x^2 + y^2)\right)\left((x^2 + y^2) - 4\right).$$

Extra credit!! c) Sketch the surface

$$z^2 = \Big(\big((x-2)^2 + y^2\big) - 1\Big)\Big(\big((x+2)^2 + y^2\big) - 1\Big)\Big(16 - (x^2 + y^2)\Big)$$