Problems, Week 5

1. a) A certain function $f(x, y)$ is known to have partial derivatives of the form

$$(*) \qquad \frac{\partial f}{\partial x} = 2xy + g(y), \qquad \frac{\partial f}{\partial y} = x^2 + 3x.$$

Please note that g is a function of y only. Use the equality of mixed partial derivatives (Clairaut's Theorem) to find the function g up to an arbitrary additive constant. Then find all functions f with partial derivatives of the form $(*)$.

b) Find all functions $f(x, y)$ satisfying

$$\frac{\partial f}{\partial x} = 2e^{2x}y + xy^2 + g(y), \qquad \frac{\partial f}{\partial y} = x^2y + 4y^3x + h(x).$$

(Hint: The solution contains four arbitrary constants.)

2. Each of four real numbers is known to lie between 0 and 50. The product of these four numbers is approximated by first rounding each number to the nearest tenth (that is, to the nearest number with one digit after the decimal point) and then taking the product of these rounded numbers. Use differentials to estimate the maximum possible error in the approximate product.

3. Consider the function $g(x, y) = x^y$.

a) How is g defined in terms of the logarithmic and exponential functions? What is the domain of g?

b) Find the partial derivatives g_x and g_y.

c) Using the fact that $2^3 = 8$, find an approximate value for $g(2.01, 3)$ from a *linearization* approximation (Stewart, p. 177). Note that you can treat y as a constant ($y = 3$), and in effect apply linearization to the function $f(x) = g(x, 3)$. Compare your answer with the result of an "exact" evaluation obtained from a calculator.

d) Similarly, find approximate and exact values for $g(2, 3.01)$.