640:251:19-20 **Problems, Week 7?** **10/17/96**

1. Leonhard Euler (1707–1783) was a great and very prolific mathematician. He published *Institutiones Calculi Differentialis* (*Methods of the Differential Calculus*) in 1755. It was an influential text, and was the first source of criteria for discovering local extrema of functions of several variables. In it Euler investigated the following specific example:

$$V = x^3 + y^2 - 3xy + \frac{3}{2}x$$

He asserted that V has a minimum both at $x = 1$ and $y = \frac{3}{2}$ and at $x = \frac{1}{2}$ and $y = \frac{3}{4}$. Was Euler correct?

Citation: *A History of Mathematics* by Victor J. Katz (New York City: Harper Collins, 1993), p. 517.

2. What straight line $y = mx + b$ best approximates the curve $y = e^x$ over the interval $0 \le x \le 1$? One way to approach this problem is to look at the error in the approximation at the point x, $\mathbf{ERR}(x) = e^x - (mx + b)$, and to make the "sum" of these errors small—where sum is interpreted as an integral. We could consider two measures of the total error:

$$\mathbf{LIN}(m,b) = \int_0^1 \mathbf{ERR}(x)\,dx \quad \mathbf{QUAD}(m,b) = \int_0^1 \big(\mathbf{ERR}(x)\big)^2\,dx.$$

a) Explain why **LIN** is not a good measure of the total error (think about cancellations of positive and negative quantities in the integral for **LIN**) and why this problem does not occur with **QUAD**. You may illustrate your explanation which should be only a few sentences long with (exaggerated) pictures.

b) Compute $\mathbf{QUAD}(m,b)$ explicitly and then find the values of m and b which minimize **QUAD**. What is this minimum value of **QUAD**?

You may use **any** computational help you wish in this problem!

c) Plot graphs of the exponential function and the "best" line whose slope and $x-$intercept were found in b) over the interval $[0,1]$.

You may use **any** computational help you wish in this problem!

This is called "least squares" fitting of a straight line to a curve.
This method is used very widely in analysis of experimental data.