640:251:19-20

Problems, Week 8

10/31/96

1. Setting up an iterated double integral can be tricky, and it's easy to produce integrals that are actually illegal. Please decide whether or not each formula below makes sense as a double integral which computes a geometric volume. For the formulas that you reject, explain your reasoning in one or two sentences; for any formulas which are legitimate, sketch the region of integration in the plane and evaluate the integral.

a) $\displaystyle\int_0^{5y}\int_0^1 x^4+y^2+7\,dy\,dx$

b) $\displaystyle\int_0^1\int_0^{5x} x^4+y^2+7\,dx\,dy$

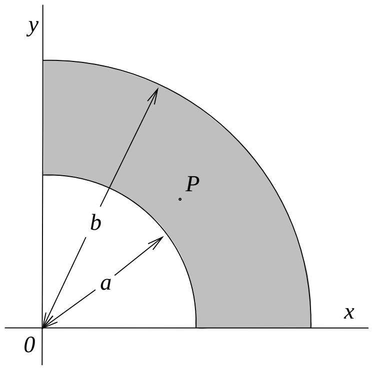
c) $\displaystyle\int_0^1\int_0^{5x} x^4+y^2+7\,dy\,dx$

d) $\displaystyle\int_1^3\int_{y-1}^{1-y^2} x^4+y^2+7\,dx\,dy$

e) $\displaystyle\int_1^{x^3}\int_{y-1}^{1-y^2} x^4+y^2+7\,dx\,dy$

f) $\displaystyle\int_0^1\int_{y-1}^{1-y^2} x^4+y^2+7\,dx\,dy$

2. a) Suppose that a and b are positive numbers with $a<b$. What are the coordinates of P, the center of mass* of the region made up of the points in the first quadrant whose distance from the origin is between a and b? Note that if the coordinates of P are $(\overline{x},\overline{y})$, then $\overline{x}=\overline{y}$ since the region is symmetric with respect to the line $y=x$: you need to compute just one coordinate of P.



b) If a region is *convex* (so any line segment whose endpoints are in the region must lie entirely within the region), then the center of mass of the region must be inside the region. But the center of mass of a non-convex region may be *outside* the region. For example, if the region illustrated above has a and b rather "large" compared to a "small" $b-a$, then P may be outside the region. Find specific numbers a and b for which this is true.

c) Now suppose that $a=1$. Find the value of b which has P sitting exactly on the inner boundary of the region.

Optional: construct a model of the region described in c). Check its properties!

* This is also called the "center of gravity" or (for a homogeneous mass distribution) the "centroid".