640:251:19-20 # Problems, Week 9 **11/7/96**

Do **not** hand in any problems, but please be sure to bring this to class tomorrow!

I want to set up a triple integral computing the mass of a solid in $\mathbb{R}^3$. The density of the solid at the point (x, y, z) is $\rho(x, y, z)$ (a function which is positive at every point) and the mass will be given by some formula like this:

$$\text{Mass} = \int_?^? \int_?^? \int_?^? \rho(x, y, z)\, d?\, d?\, d?$$

It is *your* job to choose an order of integration and the three integral signs, which *must* be chosen from among the following:

$$\int_x^{x^3} \quad \int_{x^2y^4}^{5xy^2} \quad \int_{-1}^{-2} \quad \int_{\frac{1}{x^2y^4}}^{\frac{1}{5xy^2}} \quad \int_0^{\sin(100x)} \quad \int_{-5xy}^{-(x+y)} \quad \int_1^2$$

Write the order you choose and the limits on the integrals you choose. You cannot compute the mass since I haven't told you ρ. All you can do is set up the problem, and *explain why* your choice is valid. (I think there's only one correct way to answer this problem.)

The bat and the slug

a) The average value of a function $f(x, y, z)$ over a region $\mathcal{R}$ in $\mathbb{R}^3$ is defined to be $\iiint_{\mathcal{R}} f(x, y, z)\, dv$ divided by $\iiint_{\mathcal{R}} 1\, dv$. If a hollowed–out hemisphere of inner radius A and outer radius B is defined to be the region consisting of those points in $\mathbb{R}^3$ which have third coordinate ≥ 0 and have distance to the origin $\geq A$ and $\leq B$, what is the average height (z value) of a hollowed–out hemisphere?

b) *The bat flies* ... A bat flies in and around a hemispherical cave, with water at the bottom, so it cannot land there. If the radius of the cave is R, what does the bat think that the average height of the cave is?

I have been urged by a legalistic & unpoetic colleague to add: "The bat flies totally at random throughout all of the space available to it." The colleague then remarked (while talking to me), "Of course, they've had you all semester, so they know what to expect." Hmmmm. What could she have meant?

Hint: $A = 0$ and $B = R$.

c) *The slug crawls* ... A **non-swimming** slug crawls about on the inner surface of the same cave as in b). Its motion is confined to that surface of the cave. What does the slug think the average height of the cave is?

Please add (sigh!): "The slug crawls totally at random throughout all of the space available to it."

Hint: $A \to R^-$ and $B = R$.

d) Which creature is higher (on average)?