PRACTICE PROBLEMS—EXAM 1

Please note that this is *not* a practice exam; in particular, there are more problems here than will be on the exam. Moreover, although these problems are generally similar to exam problems, it is possible that the exam will contain some problems quite different from any here. You will be allowed to bring to and use on the exam one 8 1/2 × 11 sheet of formulas and a calculator. The answers are not guaranteed.

1. Write the vector $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ in the form $\mathbf{c} + \mathbf{d}$, where $\mathbf{c}$ is a vector parallel to the vector $\mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{d}$ is a vector perpendicular to $\mathbf{b}$. $\mathbf{c}=-2(\mathbf{i}+\mathbf{j}-\mathbf{k})/3$, $\mathbf{d}=(8\mathbf{i}-7\mathbf{j}+\mathbf{k})/3$

2. Find parametric equations of the line through the point $(1, -3, 1)$ and parallel to the line of intersection of the planes $2x + 2y - z = 1$ and $3x - y = 0$. $x=t+1, y=3t-3, z=8t+1$

3. (a) Show that the line $x = 2 - t$, $y = 1 + 2t$, $z = 3 + t$ intersects the line $x = 3 + 2s$, $y = -3s$, $z = 5 + s$, and find the point of intersection.

(b) Find the equation of the plane containing both of these lines. $(1,3,4)$; $5x+3y-z=10$

4. Find the distance of the point $P(1, 2, 3)$ to the line $x = 2t + 1$, $y = 3t - 1$, $z = t - 2$. $2\sqrt{5}$

5. (a) Find the equation of the plane through the points $Q(1, 3, 2)$, $R(2, 1, 4)$, and $S(1, 5, 6)$.

(b) Find the distance of the point $P(2, 2, 3)$ to this plane.

(c) Find the angle at Q of the triangle PQR. $6x+2y-z=10$; $3/\sqrt{41}$; $\cos^{-1}(2/(3\sqrt{5}))$

6. The acceleration of a particle in the plane is $\mathbf{a} = 2\mathbf{i} + 8\mathbf{j}$.
a) Find the particle's position as a function of t if $\mathbf{v}(0)$, its velocity at time $t = 0$, is $\mathbf{i} - \mathbf{j}$ and $\mathbf{r}(0)$, its position at time $t = 0$, is $2\mathbf{i} + 3\mathbf{j}$.
b) Find the speed of the particle at time $t = 1$. $(t^2+t+2)\mathbf{i}+(4t^2-t+3)\mathbf{j}$; $\sqrt{58}$

7. Find the length of the curve $\mathbf{r}(t) = (t^2 - 1)\mathbf{i} + (3t^2 - 2)\mathbf{j} + (t^2 + 1)\mathbf{k}$ from $t = 1$ to $t = 2$. Also find the unit tangent vector to the curve. $3\sqrt{11}$; $(\mathbf{i}+3\mathbf{j}+\mathbf{k})/\sqrt{11}$

8. Find the unit tangent vector $\mathbf{T}$, the principal unit normal vector $\mathbf{N}$, and the curvature κ for the plane curve $\mathbf{r}(\mathbf{t}) = [\cos(2t) + 2t\sin(2t)]\mathbf{i} + [\sin(2t) - 2t\cos(2t)]\mathbf{j}$ for $t > 0$. $\cos(2t)\mathbf{i}+\sin(2t)\mathbf{j}$; $-\sin(2t)\mathbf{i}+\cos(2t)\mathbf{j}$; $1/(2t)$

9. Find a parameterized curve $\mathbf{r}(t)$ for which the normal component of acceleration is never zero but the tangential component is always zero.

10. Suppose $f(x, y) = (5x^2 - 2y^2)/(7x^2 + 3y^2)$ if $(x, y) \neq (0, 0)$. Find the limit of $f(x, y)$ as (x, y) approaches $(0, 0)$ along (i) the x-axis, (ii) the y-axis. Does $\lim_{(x,y)\to(0,0)} f(x, y)$ exist? Explain. $5/7$; $-2/3$; No

11. Sketch any 3 level curves of the function $f(x, y) = 2 - x^2 - y$. Label each curve with the appropriate function value.

12. Find all first and second order partial derivatives of the function $f(x, y) = x^2(y + 1) + y^2e^{2x}$.
$f_x=2x(y+1)+2y^2e^{2x}$; $f_y=x^2+2ye^{2x}$; $f_{xx}=2(y+1)+4y^2e^{2x}$; $f_{xy}=2x+4ye^{2x}$; $f_{yy}=2e^{2x}$

13. Define precisely what it means for a function $f(x, y)$ to be *differentiable* at a point (x_0, y_0). See text

14. Suppose $f(x, y) = xe^y$. Find the equation of the tangent plane to the surface $z = f(x, y)$ at the point $(2, 0, 2)$. Use differentials to find an approximation to $f(2.1, 0.2)$. $z-2=(x-2)+2y$; 2.5

15. Suppose that $w = f(x, y)$, where f is a function satisfying $f(1, 2) = 3$, $f_x(1, 2) = 1$, $f_y(1, 2) = -2$, $f_{xx}(1, 2) = 3$, $f_{xy}(1, 2) = 2$, and $f_{yy}(1, 2) = 0$. Suppose further that $x = u + v - 1$ and $y = 3uv - 1$. Find $\left.\dfrac{\partial w}{\partial u}\right|_{\substack{u=1\\v=1}}$ and $\left.\dfrac{\partial^2 w}{\partial v \partial u}\right|_{\substack{u=1\\v=1}}$. -5; 9

16. Suppose that $G(u, v)$ is a differentiable function of two variables and that $g(x, y) = G\left(\dfrac{x}{y}, \dfrac{y}{x}\right)$. Show that $xg_x(x, y) + yg_y(x, y) = 0$.

17. Find the derivative of $f(x, y, z) = \cos(xy) + e^{yz} + \ln(zx)$ at the point $P_0(1, 0, 1/2)$ in the direction of the vector $\mathbf{A} = \mathbf{i} + \mathbf{j} - \sqrt{2}\mathbf{k}$. In what direction does f decrease most rapidly at P_0? Find equations for the tangent plane and normal line to the surface $f(x, y, z) = 2 - \ln 2$ at P_0.
$3/4-\sqrt{2}$; direction of $-(\mathbf{i}+(1/2)\mathbf{j}+2\mathbf{k})$; $2x+y+4z=4$; $x=1+t, y=t/2, z=(1/2)+2t$