PRACTICE PROBLEMS—EXAM 2

Please note that this is *not* a practice exam; in particular, there are more problems here than will be on the exam. Moreover, although these problems are generally similar to exam problems, it is possible that the exam will contain some problems quite different from any here. You will be allowed to bring to and use on the exam one 8 1/2 × 11 sheet of formulas and a calculator.

The answers are not guaranteed.

1. Find the derivative of $f(x,y,z) = \cos(xy) + e^{yz} + \ln(zx)$ at the point $P_0(1,0,1/2)$ in the direction of the vector $\mathbf{A} = \mathbf{i} + \mathbf{j} - \sqrt{2}\mathbf{k}$. In what direction does f decrease most rapidly at P_0? Find equations for the tangent plane and normal line to the surface $f(x,y,z) = 2 - \ln 2$ at P_0.
$3/4-\sqrt{2}$; direction of $-(\mathbf{i}+(1/2)\mathbf{j}+2\mathbf{k})$; $2x+y+4z=4$; $x=1+t, y=t/2, z=(1/2)+2t$

2. Let $f(x,y) = 2x^3 - 6xy + y^2 + 4y$. Find all critical points of f and classify them as saddle points, local maxima, or local minima. (1,1) saddle;(2,4) local min

3. Find the absolute maxima and minima of the function $f(x,y) = x^2 - xy + y^2 - 6y$ on the closed triangular plate bounded by the lines $x = 0$, $y = 8$, and $y = x$. Min: -12 at (2,4); max: 16 at (0,8), (8,8)

4. Use Lagrange multipliers to find the maximum and minimum values of $f(x,y,z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$. 30 at $(1,-2,5)$; -30 at $(-1,2,-5)$

5. Let D be the triangular region bounded by the lines $x = 0$, $y = 0$, and $2x + 3y = 1$. Sketch the region D and evaluate the mass of a plate covering D, with density $\rho(x,y) = x$. 1/72

6. Sketch the region of integration and write an equivalent integral with the order of integration reversed for the integral $\int_0^2 \int_{x^2}^{2x} xy\,dy\,dx$. Evaluate the integral in both forms. $\int_0^4 \int_{y/2}^{\sqrt{y}} xy\,dx\,dy = 8/3$

7. Evaluate $\int_0^1 \int_y^1 \sin(x^2)\,dx\,dy$. $[1-\cos(1)]/2$

8. Change the Cartesian integral $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-(x^2+y^2)}\,dy\,dx$ into an equivalent polar integral and evaluate the polar integral. $\int_0^{\pi/2} \int_0^2 e^{-r^2} r\,dr\,d\theta = \pi(1-e^{-4})/4$

9. Find the area of one leaf of the "'rose" $r = \sin 3\theta$. $\pi/12$

10. Find the area of that portion of the "saddle" $z = 3x^2 - 3y^2$ which lies inside the cylinder $x^2 + y^2 = 4$. $\pi[(145)^{3/2} - 1]/54$

11. Express the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + 2y + z = 4$ as an iterated triple integral and then evaluate the integral. 16/3

12. (a) Let E denote the region in the first octant that is bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 9$. Express the volume of E as in iterated triple integral in (i) cylindrical and (ii) spherical coordinates. Then evaluate both integrals. $9\pi(2-\sqrt{2})$

(b) Do the same for E the entire region inside the sphere $x^2 + y^2 + z^2 = 25$ but outside the cylinder $x^2 + y^2 = 16$. 36π

13. Evaluate the integral $\int\int_D xy\,dA$ by the transformation $x = u/v$, $y = v$. Here D is the region in the first quadrant bounded by the curves $y = x$, $y = 3x$, $xy = 1$, and $xy = 3$. $2\ln 3$

14. Find the gradient vector field of the function $f(x,y) = xy$, and sketch this field in the region $-2 \le x \le 2$, $-2 \le y \le 2$. $y\mathbf{i}+x\mathbf{j}$

15. Evaluate the line integral $\int_C f\,ds$ of $f(x,y,z) = x - y + z - 2$ along the curve C given by $\mathbf{r}(t) = t\mathbf{i} + (1-t)\mathbf{j} + \mathbf{k}$ for $0 \le t \le 1$. $-\sqrt{2}$

16. Find the work $\int_C \mathbf{F} \cdot d\mathbf{r}$ done by the force field $\mathbf{F} = y^2\mathbf{i} + x^2\mathbf{j}$ on a particle that moves from $(1,0)$ to $(-1,0)$ (a) along the parabola $y = x^2 - 1$, (b) along a semi-circle centered at the origin and lying in the upper half plane, (c) along the curve given parametrically by $\mathbf{r}(t) = \langle t^2 - 3t + 1, t(2-t)\rangle$, $0 \le t \le 2$. $-16/15$, $-4/3$, $-32/15$