311:09 **Exam #1** **10/6/94**

Part I

No books or notes may be used in this part.

1. Prove that $2n - 3 \leq 2^{n-2}$ for all $n \geq 5$, $n \in \mathbb{N}$.

2. Prove using the definition of limit that

$$\lim_{n \to \infty} \left(\frac{1 + 2n^2}{3 + 4n^2} \right) = \frac{1}{2}.$$

Part II

Do all problems in this part. Use any books and notes that you wish.

3. Suppose $a, b \in \mathbb{R}$ and suppose that for every $\varepsilon > 0$ we have $a \leq b + \varepsilon$.

a) Prove that $a \leq b$.

b) Show by example that it does *not* follow that $a < b$.

4. If A and B are any nonempty sets of numbers which have the property that $a \leq b$ for all $a \in A$ and all $b \in B$, prove:

a) $\sup A \leq b$ for all $b \in B$.

b) $\sup A \leq \inf B$.

5. If $T := \left\{ \frac{1}{n} + \frac{1}{m} : n, m \in \mathbb{N} \right\}$, find $\inf T$ and $\sup T$. Prove that your answers are correct.

6. Prove that if $\lim_{n\to\infty}(x_n) = x$ and if $x > 0$, then there is a positive integer M such that $x_n > 0$ for all $n \geq M$.

7. If $a, b \in \mathbb{R}$ and $b \neq 0$, prove that $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$.

8. Suppose (a_n) and (b_n) are any sequences of real numbers with the following properties:

(a_n) converges, and $\lim_{n\to\infty}(a_n)$ is 0.

(b_n) is bounded above by 3 and below -3: so $|b_n| \leq 3$ for all $n \in \mathbb{N}$.

Prove that the sequence (c_n) with c_n defined by

$$c_n := a_n \cdot b_n$$

converges, and that $\lim_{n\to\infty}(c_n) = 0$.