Report on a section of Math 311

given Fall, '94, by S. Greenfield

I've avoided teaching Math 311 for several years (since, in fact, the Fall, '90, semester). My experiences with the course were not good. I thought that I worked hard preparing lectures, was diligent in grading homework, and was available quite a bit for outside help. Our students cherish the tradition that math courses should not involve much actual thought, but should be only superficially algorithmic ("Whaddya mean the derivative is stuff with lim and $\triangle x$? I *know* the definition of derivative — it's nx^{n-1}."). They do it so convincingly, so persistently, that they frequently cause their instructors to agree, at least in terms of such behavior as standards for getting grades above D.

But Math 311 is an effort to give students the logical foundations of analysis. It proves the sequence of steps going from the axioms of a complete ordered field to the basic properties of continuous and differentiable functions, of derivative and integral of one-variable functions. Teaching it has been depressing if student knowledge of these ideas is what's wanted. The instructors must fight against the unintellectual spirit of the students, and must also change the students' entire view of what's worthwhile in math courses both by persuasion and by changing the reward structure. That is, the circled answer of 17π surrounded by a scribbled mass of computation isn't the aim: the aim is a careful exposition of a mathematical argument.

I taught the course in 1990, as I said. I didn't think I was very effective. I knew I needed more time. I tried to institute a voluntary extra meeting each week to go over problems (a practice which Larry Corwin did at approximately the same time). Some students couldn't attend the extra session at all since the Rutgers undergraduate schedule is more varied and more erratic than any butterfly's flight in spring. And of course such a session could be no more than voluntary without any official changes countersigned by the school. Most importantly, there was no evidence that a traditional problem session once a week did any good.

Last year Mike Beals decided it was time to do something about the dreadful gap between what we wanted students to learn in Math 311 and what they usually got from the course. He suggested that an additional 80-minute mandatory workshop be added to the course, that this time be used for small group work on interesting problems (imitating the success of EXCEL), and that the students be requested to hand in carefully written workshop problems each week. His request was supported by funding from Susan Forman's office, and, more importantly, by Charlie Sim's obtaining various important FAS and scheduling permissions. I wanted to try to teach Math 311 in the new format, and I hoped that it would make a significant difference in student performance.

Here are some factors which might have influenced student performance, both this year and in comparison to previous years' instantiations of the course.

- I taught the evening section. In previous years, I taught sections given during the day. In previous years, students attended many office hours. Although I made myself available before and after class, this isn't the same as having a quiet block of time in which one or two students could interact with me, discussing the subject matter of 311. I had remarkably few students visit me in my office during the day. Some students worked full-time, and were classic university college students. More of them just seemed to forget about the course during the day. In lower level courses in recent years, I have used electronic mail to diminish the barriers of the Rutgers campus and the Rutgers schedule. The expertise and enthusiasm in using e-mail that first and second year students have displayed to me did not generally extend to these 311 students. I got e-mail from only a handful of the students, and almost all of the messages I got were from one student. I was disappointed. By the way, at the first two meetings of the course, I encouraged students to give me their names, local 'phone numbers and e-mail addresses. I used this information compile a list to give out to everyone in the course. A copy of my hand-out asking for the information is appended.

- I usually know when I'm giving "good" lectures (although I still can deceive myself). I usually know when an office hour visit has been useful for a student, or when a written homework set has accomplished what I want it to. But . . . I was uneasy with the workshops, even though I had some previous experience with EXCEL and its workshops. These are different students, older and in almost all cases less enthusiastic. I didn't know how much help to extend. I didn't know when to stop nudging. I walked around a great deal during the first two or three workshops, almost always talking. Later I sat down in front of the room the greater part of the time, pretending to grade homework but in fact monitoring the discussion of the groups. I would sometimes try to get them back on task if they were spending too much time (in my estimation) discussing how hard the latest 477 exam was. Or I would look up and tell one or more people in a group that their approach was a good one. Or I would try to give more-or-less cryptic hints. But ... mostly I learned *not* to intervene. This was difficult. It was a new mode of instruction in an upper level course for me. See the comment below, though.

- We asked too much in this course. The transition between Math 311 with workshops and previous math courses was great. I don't think that one course alone is enough to overcome the difference between passive and active interaction with mathematics.
- My workshop occurred after one of my lectures. Thus formal class time stretched from 6:10 until 9:00 PM on Tuesday night. This was a very, very long time. I think that evening 311's and 351's with workshops should have the classes on three evenings. After talking with students on Tuesday after class, I often did not leave campus until after 9:30.
- I started out (for the first three weeks or so) doing all the grading myself. I was intent upon fast turnaround, with ample remarks on all papers. Wow — what a lot of time that took! I found J. Dare, one of our grad students, who was willing *and* able to grade the textbook problems (not at all trivial in the amount of expertise needed!). I had students hand in workshop problems one week after each workshop, on Tuesday. I had them hand in textbook problems on Thursday. Keeping up with the grading was difficult. I did **not** ask students to redo workshop problems, as had been one of the rules in the initial proposal of the experiment. I could barely keep straight the rush of assignments myself. At the end of each workshop I decided how many (usually one or two, sometimes unspecified, sometimes with choices [e.g., one from problems 1 and 2 and one from problems 3 and 4]) were to be written up. I graded the workshop problems 50% on mathematical content and 50% on exposition, and I tried to grade honestly. I handed out written solutions to all homework exercises for the first three weeks, but that took an enormous amount of time. Thereafter, I handed out written solutions only when I couldn't get graded homework back to students before an exam. I used workshop time to prepare for the two in-class exams by giving out exams from a previous course. I used workshop time to have students, near the end of most workshops, come up to the front of the room and talk about how to do a problem or part of a problem. I don't feel I found optimal ways of using the workshop times.

I think that we ought to continue the "experiment" and we ought to see how to make the workshops work as well as they can. I know that I would like to try again, and I also know that I would definitely do the following:

> I would ask to be observed and helped in a workshop. I would like to observe and help in the workshop of another instructor. I'd like to figure out what "works" and what doesn't, and maybe even why (if possible).

Here are the results of each exam:

Warning The grades in this exam are out of a total of 80 points!

Exam 1: mean 40.03, standard deviation 15.13, median 41, high 66, low 20; # of students taking exam: 17. A (≥ 70): 0, B+ (> 70 & ≥ 65): 4, B (> 65 & ≥ 60): 0, C+ (> 60& ≥ 55): 2, C (> 55 & ≥ 45): 0, D (> 45 & ≥ 40): 3, F(> 40): 8.

The grades were *dreadful* on this exam. I copied a format Mike Beals had used the previous year when he was teaching Math 311. He gave a few questions in usual "closed-book" format, and the remainder of the exam in "open-book" format. My students did dreadfully. They had not internalized the definitions, and almost every student copied inappropriate passages from the text onto their exam papers, many inserting errors and misstatements of their own devising in these excerpts. I did not think that the exam would be so difficult. I corrected the exam, and thought that students needed another chance — that perhaps I had not prepared them adequately. I therefore (slightly) trashed my initial elaborate grading scheme. I told them to take the corrected exam (with ample hints!) home and do it again, and that this effort would be averaged in with the other in-class exams (not with the final!). So here is the result of the second attempt.

Warning The grades in this exam are out of a total of 80 points!

Exam 2: mean 64.41, standard deviation 14.46, median 73, high 80, low 26; # of students taking exam: 17. A (≥ 70): 9, B+ (> 70 & ≥ 65): 3, B (> 65 & ≥ 60): 1, C+ (> 60& ≥ 55): 1, C (> 55 & ≥ 45): 1, D (> 45 & ≥ 40): 2, F(> 40): 0.

I encouraged the students strongly, and asked that this "take-home" exam be the results of their own efforts. I believe that in this, as in almost all the work students turned in, high standards of academic honesty were maintained. I compliment the students on this quality, and I believe it was an echo of my own strongly stated desires that I would invest a large portion of my own time in the course, and would try to give them the best opportunity I could to learn the material and display their mastery. Considering all this, and considering that the students had a graded version of their work on the problems to start from, and considering that most of the problems were homework problems or close to them and that the students had both written solutions to the homework problems and graded results of their efforts on the homework problems, their performance on this exam was in some cases ludicrously bad. For example, there was one score of 26 out of 80 on exam 2!

Exam 3: mean 57.35, standard deviation 19.05, median 63, high 88, low 16; # of students taking exam: 17. A (≥ 80): 2, B+ (> 80 & ≥ 75): 2, B (> 75 & ≥ 65): 1, C+ (> 65& ≥ 60): 3, C (> 60 & ≥ 55): 4, D (> 55 & ≥ 50): 1, F(> 50): 4.

The grades on the this exam were more what I expected. I allowed students to have notes written on one sheet of paper (both sides, if desired). I did the same on the final, by the way. In some sense, this worked against them, because I therefore did not ask them to state a string of definitions on the exam. But on balance I believe possession of a "crib sheet" made students feel more secure and that they benefitted from its creation. Unlike the open-book policy on exam #1, I had less parroting (repeating without any real knowledge) of phrases in math language. Preparation of the crib sheet also made them concentrate on what was important in the course and its creation was therefore a valuable study aid.

Warning The grades in this exam are out of a total of 200 points!

Final exam: mean 115.38, standard deviation 40.74, median 126.5, high 191, low 32; # of students taking exam: 16.
A (≥ 160): 2, B+ (> 160 & ≥ 150): 2, B (> 150 & ≥ 130): 1, C+ (> 130& ≥ 120): 3, C (> 120 & ≥ 110): 4, D (> 110 & ≥ 100): 1, F(> 100): 4.

One student who would most likely have gotten an F did not take the final exam.

I gave final grades for the course using criteria very close to those I initially told students. I weighted the final exam 35%. I weighted the three other exams equally and counted them for 35% of the final grade. I counted the text homework and the workshop problems 15% each. The out-of-class portions of the grades were generally remarkably consistent with in-class measurements. My final grades in the course include one TF, a student who did *much* more poorly on the final than previous work would have suggested. Reasonably, I would have expected at worst a D, most likely a C, as a result of this student's efforts.

I was rather strict in making the workshop and text problems *count*, even though it adversely affected the grades of some students. I should remark that the work on the problems also helped the grades of other students. These parts of the course took a great deal of effort on the part of students, and I wanted that effort to show.

Here are my grades in the course:

Final grades: A: 1, B+: 2, B: 5, C+: 1, C: 2, D: 1, F: 4; TF: 1 (likely to be a C or a D).

There were 17 students who received a final grade in this class.

By comparison here are the final grades for the section of Math 311 I taught in Fall, 1990:

Final grades: A: 1, B+: 1, B: 2, C+: 0, C: 2, D: 3, F: 5.

There were 14 students who received a final grade in this class.

Please note that in both courses, a number of students equal to approximately an additional 50% of the class tried the course: that is, they sat in at least one lecture (usually several), frequently handed in some homework, talked to me, and sometimes even stayed for an exam. But they dropped out. Again: the proportion of "dropouts" was similar in '90 and '94.

Comments on the final grades If A= 4.0 and F= 0, then the average grade in '90 was $\boxed{1.5}$, about a D+ (if there were such) while the average grade in '94 was $\boxed{2.06}$, a bit better than a C. Thus the experiment was a **success** in a very measurable way. I am very, very surprised. As the semester progressed, I grew more and more despondent. It seemed that the students were unenthusiastic and not really learning and doing as much as we had hoped the workshop experiences would help them to. But I (nearly reluctantly!) believe that the numerical difference in grades noted above *is* significant. Students did do better on average in '94. They still were far, far less accomplished than we hoped. This gulf between our hopes and what the students were able to do may be impossible to bridge with the background of the 8 to 10 earlier math courses they take at Rutgers, courses which for C or even B work ask for little real thought. The workshops with their additional class time, with the additional small group work, with the additional graded homework, did lead to improved grades. I cannot testify that student writing skills (especially technical writing) necessarily improved, although I suspect they did. I don't feel that I have numbers to substantiate such a claim. But I do think that the course grades, which reflected many measurements of achievement in both courses (the '90 version of 311 had lots of standard graded homework), represent a distinct increase in proficiency in doing similar 311 exams. Whether the additional class time, grading expense, and work for the students and instructors is *WORTH* this difference is a serious policy question. I personally think that whatever the decision will be on this question, we *must* make our whole sequence of math major courses more intellectually respectable, so that Math 311 is no longer one of the few intellectual pinnacles jutting up from a plain of pebbles of trivial computations*.

What are all the crazy things that follow?

Various handouts to the Math 311 class

Page #'s refer to hand-written numbers in the upper right-hand corners of the pages

* Bad metaphor winner of the essay, surely!

** In an effort to stimulate interest, I asked for a random polynomial, and proceeded to prove that its iterates, with any starting point on the unit interval, converged to a unique real number. Of course, as students suggested coefficients and degrees for the polynomial, I "cleverly" wrote them down to assure that the resulting function was a contraction mapping. Oh well, I thought it was a cute idea.

† I could either teach the syllabus or teach the students. I decided to try to do the latter. This meant that I needed to adjust what I would cover several times during the semester. At the end, I had to make a decision about what material about the derivative and integral I would try to cover. Incidentally, I was not able to do all I wanted even in the last few meetings: more "slippage" occurred.