(20) 1. Suppose that $\mathbf{A}$ is the vector $2\,\mathbf{i}+3\,\mathbf{j}+\mathbf{k}$ and that $\mathbf{B}$ is the vector $\mathbf{i}-\mathbf{j}+2\,\mathbf{k}$ and that P is the point $(1,1,2)$.
a) Find an equation of the plane containing P which is parallel to both $\mathbf{A}$ and $\mathbf{B}$.
b) What is the angle between $\mathbf{A}$ and $\mathbf{B}$?

(20) 2. Suppose $w=x^2e^{z+2y}$
a) Compute w, w_x, w_y, and w_z when $x=1$, $y=-1$, and $z=2$.
b) Suppose that x changes from 1 to 1.02, y changes from -1 to -1.03, and z changes from 2 to 2.04. Use linear approximation to predict the approximate change in w.

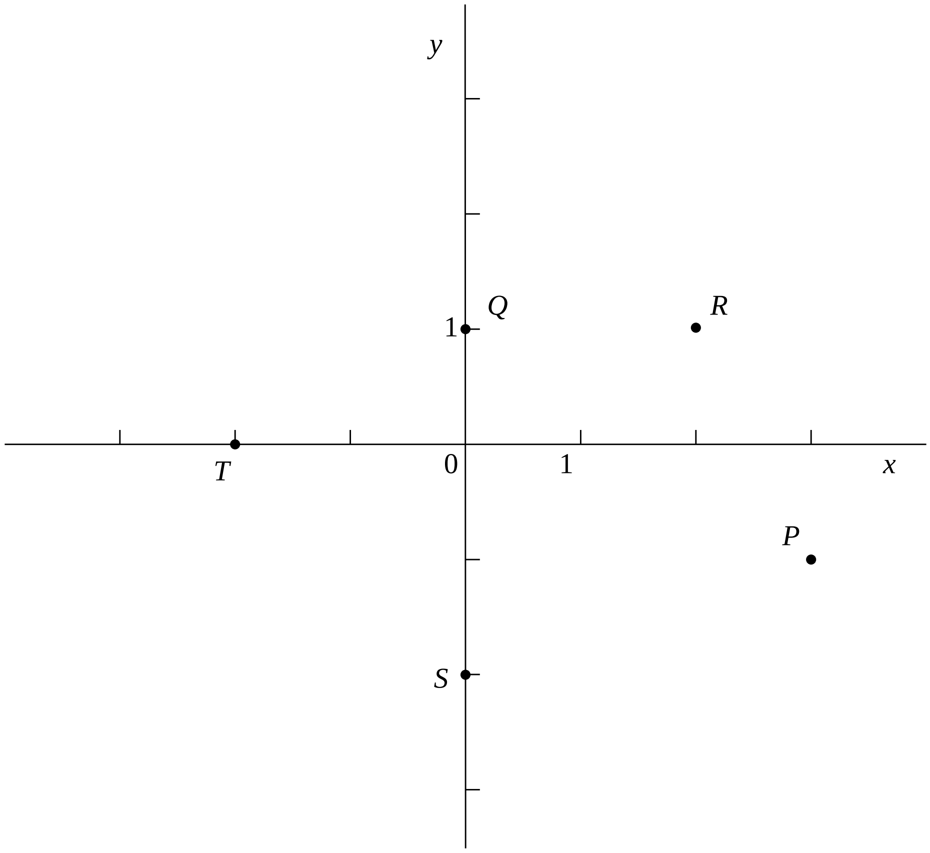
(15) 3. Find the critical points of the function $f(x,y)=xy-x^2-2y^2-3x-2y+4$. Decide whether each critical point is a local maximum, local minimum, or saddle point.

(15) 4. Find the maximum and the minimum values of $g(x,y,z)=x-y+3z$ subject to the constraint $x^2+y^2+4z^2=4$.

(20) 5. Sketch the three level curves of the function

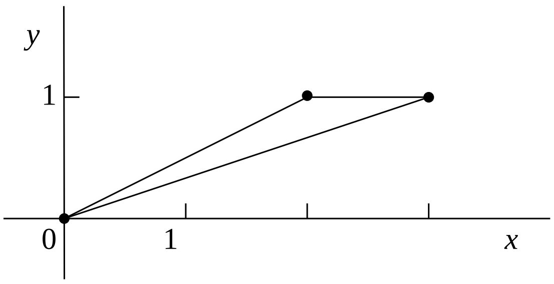
$$W(x,y)=\frac{x^2-4y^2}{10}$$

which pass through the points $P=(3,-1)$ and $Q=(0,1)$ and $R=(2,1)$. Be sure to label each curve with the appropriate function value and be sure that your drawing is clear and unambiguous.
Also sketch on the same axis the vectors of the gradient vector field ∇W at the points P and Q and R and S and T. The point S is $(0,-2)$ and the point T is $(-2,0)$.



(20) 6. Suppose that D is the triangular region in the plane with corners $(0,0)$ and $(3,1)$ and $(2,1)$, with straight line segments as sides.

Compute $\displaystyle\iint_D x^3y\,dA$.



(15) 7. Suppose $J = \displaystyle\int_0^1 \int_0^{\pi} \int_0^{\sqrt{1-z}} (z^2)\, r dr\, d\theta\, dz$ (a triple integral in cylindrical coordinates).
a) Sketch the solid region in $\mathbb{R}^3$ over which the integral J is computed, and accompany your sketch with equations describing the boundary surfaces of the region using the standard rectangular coordinates x, y, and z.
b) Compute J.

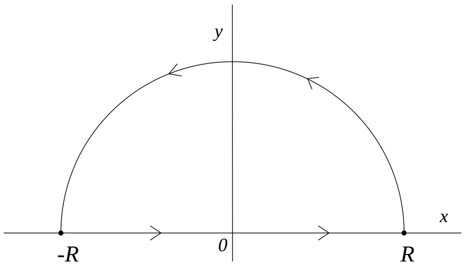
(15) 8. Use spherical coordinates to evaluate the triple integral $\displaystyle\iiint_E \left(x^2+y^2+z^2\right)^{-5/2}\,dV$, where E is the region in space ($\mathbb{R}^3$) outside the sphere of radius 1 centered at the origin.

(15) 9. a) Suppose $\mathbf{H}(x,y,z) = (2xz + \sin y)\,\mathbf{i} + x\cos y\,\mathbf{j} + x^2\,\mathbf{k}$. Find a function h such that $\mathbf{H} = \nabla h$.
b) Use your answer to a) to evaluate $\displaystyle\int_C \mathbf{H}\cdot d\mathbf{r}$ where C is the curve $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k}$, $0 \le t \le 2\pi$.

(20) 10. Suppose that V_R is the closed curve consisting of the straight line segment from $(-R,0)$ to $(R,0)$ followed by the upper semicircle (of radius R and center $(0,0)$) joining $(R,0)$ to $(-R,0)$. Find the value of R so that

$$\int_{V_R} \left(xy + x^3\cos(x^2)\right)dx + \left(3xy^2 + x^3 + 5e^{(5y^2)}\right)dy = 5$$

Hint Green's Theorem.



(25) 11. In this problem, the vector field $\mathbf{F}(x,y,z)$ is $x^2\,\mathbf{i} + (3y+z)\,\mathbf{j} + xz\,\mathbf{k}$.
a) Compute div $\mathbf{F}$.
b) Suppose U is the unit cube. That is, U is a cube with all edges 1 unit in length, in the first octant with sides parallel to the coordinate planes, and having the origin equal as one corner. U's vertices (its corners) are $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, $(0,0,1)$, $(1,1,0)$, $(1,0,1)$, $(0,1,1)$, and $(1,1,1)$.
Compute $\displaystyle\iiint_U \text{div } \mathbf{F}\,dV$.
c) Compute the total outward flux of F over the surface of U; do *not* use the Divergence Theorem, but use the definition of flux.