Mathematics 251: Lab 2 QUADRIC SURFACES

A quadric surface is the graph of a second-degree equation in three variables x, y, and z. The most general form of such an equation is:

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0,$$

where $A, B, C, D, E, F, G, H, I$, and J are constants.

In this lab, we use Maple to help visualize some types of quadric surfaces that can arise from equations of the above form.

Please turn in only the printout of your Maple worksheet. Include explicit answers to all questions asked, using the **text** feature of Maple to insert them in the worksheet (don't write them by hand). Insert graphs directly into your worksheet; instructions were given in the initial handout. Remove from the worksheet any extraneous material such as the output of commands like `with(plots);` and any errors you have made.

1. The surface

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

is called an elliptic paraboloid.

a) Use the `plot3d` command to plot the elliptic paraboloid corresponding to $a = 2$, $b = 3$, $c = 5$. One way to do this is to first define constants a, b, and c by typing

```
a:=2; b:=3; c:=5;
```

To plot the elliptic paraboloid, then type

```
plot3d(c*(x^2/a^2 + y^2/b^2), x= -3..3, y=-3..3);
```

Also obtain a plot in which you include the option `axes=BOXED` by typing

```
plot3d(c*(x^2/a^2 + y^2/b^2), x= -3..3, y=-3..3, axes=BOXED);
```

Note that if you forget to include this option in the `plot3d` command, you can also obtain the option after Maple draws the plot by clicking the mouse on **Axes**, then dragging the mouse down, releasing on **Boxed**. Clicking the middle mouse button will then cause Maple to redraw the plot using the Boxed option.

b) Use Maple to help determine the curve formed by the intersection of the elliptic paraboloid of part (a) with the plane $z = 4$ by typing

```
plot3d({c*(x^2/a^2 + y^2/b^2), 4}, x= -3..3, y=-3..3, axes=BOXED);
```

Note the use of the braces {} when plotting more than one function. Determine without the use of Maple the equation of the curve of intersection and use the **text** feature of Maple to enter it into your Maple worksheet, making sure to label your equation (e.g., Equation of curve of intersection). Also state explicitly what type of curve this is.

The surface

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

is called a hyperbolic paraboloid.

c) Plot the hyperbolic paraboloid corresponding to $a = 2$, $b = 3$, $c = 5$.

d) Obtain a plot which shows the intersection of this paraboloid with the plane $z = 4$. As in 1b, clearly state the equation and the type of the curve is formed by this intersection.

2. An ellipsoid is a quadric surface of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \qquad (*)$$

where a, b, and c are positive constants.

One approach to plotting such a surface is to solve for z, obtaining the two surfaces

$$z = c\sqrt{1 - x^2/a^2 - y^2/b^2}, \qquad z = -c\sqrt{1 - x^2/a^2 - y^2/b^2}.$$

and then to plot both of these. Here we take an alternate approach.

a) Use the `implicitplot3d` command to plot the ellipsoid corresponding to $a = 2$, $b = 3$, $c = 5$. To activate this command, you first need to type `with(plots):`.

```
a:=2; b:=3; c:=5;
with(plots);
implicitplot3d(x^2/a^2 + y^2/b^2 + z^2/b^2=1,x=-2..2,y=-3..3,z=-5..5);
```

Also obtain a plot showing the true shape of the ellipsoid by adding the option `scaling=CONSTRAINED` or choosing this option from the **Projection** menu in the plot already produced. Include both plots in your worksheet.

b) Use Maple to help determine the curve formed by the intersection of the ellipsoid of part (a) with the plane $z = 4$ by typing

```
implicitplot3d({x^2/a^2 + y^2/b^2 + ^2/c^2=1,z=4}, x = -2..2,
y = -3..3, z = -5..5);
```

As in 1b, use the **text** feature of Maple to give the equation and type of the curve of intersection.

c) Obtain a plot which clearly shows the curve formed by the intersection of the ellipsoid of part (a) with the plane $x = 1$; include a statement of the equation and type of this curve.

3 a) Use the `plot3d` command to plot the surfaces

$$z = x^2 + y^2, \qquad z = 1 - y^2$$

on a common screen using the domain $|x| \leq 1.2$, $|y| \leq 1.2$.

b) It is often useful to have a plot of the projection of the curve of intersection of two surfaces onto one of the coordinate planes. This can be easily done using the `implicitplot` command.

Obtain a plot of the projection on the xy plane of the intersection of the surfaces of part (a) by typing

```
with(plots):
implicitplot(x^2+y^2 = 1-y^2,x=-1.2..1.2,y=-1.2..1.2);
```

c) To see the real shape of the curve, obtain a plot adding the option `scaling=CONSTRAINED` to your command, i.e., type

```
implicitplot(x^2+y^2 = 1-y^2,x=-1.2..1.2,y=-1.2..1.2,scaling=CONSTRAINED);
```

d) Without using Maple, determine the equation of the curve you have plotted in part (c) and enter it into your Maple worksheet using the **text** feature.