640:251:01-03 **a sort of** **Diagnostic Exam** **1/22/97**

Do all of these problems. Please work in groups of 3 or 4 people. Check your answers. Talk to each other. Use your textbook and calculator. Hand in legible solutions to all the problems at the end of the period. The purpose of these problems is to help the instructors tell if you are ready for this course. Please try to answer the questions correctly and be sure to print your name clearly on your answer page(s).

1. Sketch the curves $x = e^{ky}$ on the same axes for $k = 0$, $k = +3$, and $k = -3$. Be sure to label which value of k corresponds to which curve.

2. Sketch the curves $1 = kx^2 + y^2$ on the same axes for $k = 0$, $k = +3$, and $k = -3$. Be sure to label which value of k corresponds to which curve.

3. Suppose $f(x) = .04(x^2 - 3)e^{-.5x}$.

a) Where is f zero?

b) Does f have any local extreme values? If yes, find them and describe what kind of extreme values they are.

c) Does f have any inflection points? If yes, find them.

d) Draw a rough sketch of the graph of $y = f(x)$. Label any interesting features you have discovered. What can you say about any asymptotic behavior of the graph?

4. Point A is five miles north and two miles west of point B. Point B is three miles south and seven miles east of point C. Point D is eight miles west of point C. Assuming all of these points are on a flat plane, what is the distance from D to A?

5. In this problem and in the whole course, the two-dimensional vector $< 1, 0 >$ will be called $\mathbf{i}$ and the two-dimensional vector $< 0, 1 >$ will be called $\mathbf{j}$.

> We say that two vectors have *the same direction* if each of them is a **positive** scalar multiple of the other. For example, the two vectors $\mathbf{i}+\mathbf{j}$ and $3\mathbf{i}+3\mathbf{j}$ <u>do</u> have the same direction (the positive scalar is either 3 or $\frac{1}{3}$ depending on your point of view!), while the two vectors $\mathbf{i}+\mathbf{j}$ and $3\mathbf{i}+\mathbf{j}$ do <u>not</u> have the same direction, and the two vectors $\mathbf{i}+\mathbf{j}$ and $-2\mathbf{i}-2\mathbf{j}$ also do <u>not</u> have the same direction.

a) Suppose $\mathbf{a}$ and $\mathbf{b}$ are two-dimensional vectors, and you know that $|\mathbf{a}| = 8$ and $|\mathbf{b}| = 2$. Draw a picture of suitable vectors so that $|\mathbf{a}+\mathbf{b}| = 5$ **and** the vector $\mathbf{a}+\mathbf{b}$ has the same direction as $\mathbf{i}-\mathbf{j}$.

b) Suppose $\mathbf{a}$ and $\mathbf{b}$ are two-dimensional vectors, and you know that $|\mathbf{a}| = 8$ and $|\mathbf{b}| = 2$. Draw a picture of suitable vectors so that $|\mathbf{a}+\mathbf{b}|$ is as *large* as possible **and** the vector $\mathbf{a}+\mathbf{b}$ has the same direction as $\mathbf{i}-\mathbf{j}$.

c) Suppose $\mathbf{a}$ and $\mathbf{b}$ are two-dimensional vectors, and you know that $|\mathbf{a}| = 8$ and $|\mathbf{b}| = 2$. Draw a picture of suitable vectors so that $|\mathbf{a}+\mathbf{b}|$ is as *small* as possible **and** the vector $\mathbf{a}+\mathbf{b}$ has the same direction as $\mathbf{i}-\mathbf{j}$.