640:251:01-03 # Workshop Problems **1/29/97**

We'll tell you one problem whose solution should be written up for next week's meeting. Your written solution should be of high-quality, with complete explanations given including any useful diagrams or equations. While you are encouraged to discuss the problems with other students and with the instructors, your written work must be your own explanation of the problem.

1. Suppose $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^3$, and the following is known:

$$5 \leq \|\mathbf{v}\| \leq 10, \quad 30 \leq \|\mathbf{w}\| \leq 40, \quad 30^\circ \leq \theta \leq 60^\circ$$

where θ is the angle between $\mathbf{v}$ and $\mathbf{w}$. Then find appropriate over– and under–estimates for the following real numbers using all of the information given:

$$\mathbf{v} \cdot \mathbf{w}, \quad \|\mathbf{v} \times \mathbf{w}\|, \quad \|\mathbf{v} + \mathbf{w}\|$$

2. Let L be the line with vector equation $\mathbf{r} = \langle 1, 0, 3 \rangle + t\langle -1, 2, 1 \rangle$. Find equations for two orthogonal planes containing L, one of which passes through the origin. (Hint: find the equations one at a time).

3. Find the distance between the pair of skew lines given below ("skew" in this case means a bit more than non-intersecting – it means that they are non-intersecting *and* not parallel):

$$\text{The line } L_1 \text{ is : } \begin{cases} x = 2t + 1 \\ y = -t - 1 \\ z = 3t\,^{*} \end{cases} \qquad \text{The line } L_2 \text{ is : } \begin{cases} x = 3s + 2 \\ y = 5s - 2 \\ z = -4\,^{**} \end{cases}$$

Comments: I used different letters (t and s) for the parameters in the two lines. This was solely to help you, because these letters are "dummy" variables (similar in logical status to the letters in a definite integral – surely $\int_0^1 w^2 dw$ and $\int_0^1 u^2 du$ are the same). How do I know that these lines are indeed "skew"? At least, is there an easy way to check if they are parallel?

* O.k., I mean $3t + 0$.
** O.k., I mean $0s - 4$.