640:251:01-03 # Workshop Problems **2/12/97**

We'll tell you one problem whose solution should be written up for next week's meeting. Your written solution should be of high-quality, with complete explanations given including any useful diagrams or equations. While you are encouraged to discuss the problems with other students and with the instructors, your written work must be your own explanation of the problem.

1. a) Consider the function $h(x,y) = x^y$. What is the domain of h? Find the first partial derivatives of h. I know that $2^3 = 8$. If x is increased by .01 (so (x,y) changes from $(2,3)$ to $(2.01,3)$), what change in h would you expect using the linearization* approximation? What happens if y is increased by .01, leaving x fixed at the original value of 2? Please compare the "exact answers" to the linearization answers.

b) Consider the function $j(x,y,z) = x^{(y^z)}$. What is the domain of j? Find the first partial derivatives of j. I know that $2^{\left(3^4\right)} \approx 2.417 \cdot 10^{24}$** If we "kick" one of the variables by +.01, which changed variable will likely make the biggest change in the value of j? Support your assertion by a simple linearization argument based on the derivatives you've calculated.

2. a) Sketch the surface

$$x^2 + y^2 + z^2 = 1.$$

To help with part b) below, think of this as sketching solutions to the equation $z^2 = 1 - (x^2 + y^2)$, and remember that squares must be non-negative. Where is the right-hand side zero, and what is z then? How many values of z are there when the right-hand side is positive?

b) Sketch the surface

$$z^2 = \left(1 - (x^2 + y^2)\right)\left((x^2 + y^2) - 4\right).$$

c) Sketch the surface

$$z^2 = \Big(\left((x-2)^2 + y^2\right) - 1\Big)\Big(\left((x+2)^2 + y^2\right) - 1\Big)\Big(16 - (x^2 + y^2)\Big)$$

Note: Explain your pictures as well as you can! You may be able to draw these with a program, but you can also draw them with "pure thought". Or explain them that way.

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* Stewart, page 177

** Actually, it is *exactly* 24178 51639 22925 83494 12352. Computers are wonderful toys when idle minds are playing. Amazingly, this is not Avogadro's number.