640:251:01-03 **Workshop Problems** **3/5/97**

1. What straight line $y = mx + b$ best approximates the curve $y = e^x$ over the interval $0 \le x \le 1$? One way to approach this problem is to look at the error in the approximation at the point x, $\mathbf{ERR}(x) = e^x - (mx + b)$, and to make the "sum" of these errors small—where sum is interpreted as an integral. Here we consider two measures of the error:

$$\mathbf{LIN}(m,b) = \int_0^1 \mathbf{ERR}(x)\,dx \quad \mathbf{QUAD}(m,b) = \int_0^1 \big(\mathbf{ERR}(x)\big)^2\,dx.$$

a) Explain why **LIN** is not a good measure of the total error (think about cancellations of positive and negative quantities in the integral for **LIN** and about whether there *is* a nice "minimizer" using **LIN**'s value as the criterion) and explain why this problem does not occur with **QUAD**. You may illustrate your explanation which should be only a few sentences long with (exaggerated) pictures.

b) Compute $\mathbf{QUAD}(m,b)$ explicitly and then find the values of m and b which minimize **QUAD**. What is this minimum value of **QUAD**?

You may use **any** computational help you wish in this problem!

c) Plot graphs of the exponential function and the "best" line whose slope and $x-$intercept were found in b) over the interval $[0,1]$.

You may use **any** computational help you wish in this problem!

This is called "least squares" fitting of a straight line to a curve.
This method is used very widely in analysis of experimental data.

2. a) What is the maximum of the function $f(x,y) = 3x + 5y$ subject to the constraint $x^2 + y^2 = 1$, and where is it attained? Draw a picture of the constraint and of some level lines of f which illustrate the method of Lagrange multipliers and show where the maximum is attained.

b) What is the maximum of the function $f(x,y) = 3x + 5y$ subject to the constraint $x^{10} + y^{10} = 1$ and where is it attained? Find your result exactly in terms of known or computable values of familiar functions and then give numerical approximations of your answers. Draw a picture as in a).

c) What is the maximum of the function $f(x,y) = 3x + 5y$ subject to the constraint $x^{1/10} + y^{1/10} = 1$, and where is it attained? Find your result exactly in terms of known or computable values of familiar functions and then give numerical approximations of your answers. Draw a picture as in a).

d) What do you think happens to the maximum of the function $f(x,y) = 3x + 5y$ subject to the constraint $x^n + y^n = 1$ and to where it is attained when $n \to +\infty$? Draw a picture.

e) What happens to the maximum of the function $f(x,y) = 3x+5y$ subject to the constraint $x^n + y^n = 1$ and to where it is attained when $n \to 0^+$? Draw a picture.