640:251:01-03 # Workshop Problems **3/12/97**

1. **Syntax** Setting up an iterated double integral can be tricky, and it's easy to produce integrals that are actually illegal. Please decide whether or not each formula below makes sense as a double integral which computes a geometric volume. For the formulas that you reject, explain your reasoning in one or two sentences; for any formulas which are legitimate, sketch the region of integration in the plane and evaluate the integral.

a) $\int_0^{5y} \int_0^1 x^4 + y^2 + 7 \, dy \, dx$ b) $\int_0^1 \int_0^{5x} x^4 + y^2 + 7 \, dx \, dy$

c) $\int_0^1 \int_0^{5x} x^4 + y^2 + 7 \, dy \, dx$ d) $\int_1^3 \int_{y-1}^{1-y^2} x^4 + y^2 + 7 \, dx \, dy$

e) $\int_1^{x^3} \int_{y-1}^{1-y^2} x^4 + y^2 + 7 \, dx \, dy$ f) $\int_{-1}^1 \int_{y-1}^{1-y^2} x^4 + y^2 + 7 \, dx \, dy$

Impropriety Improper integrals arise very often in calculations, usually because they are easier to compute than proper ones (!) or because they are helpful approximations (since electrons and atoms are very, very small). Integrals are *improper* if the integrand is unbounded (i.e., goes to $\pm\infty$) in the integration region or if the integration region is unbounded. We'll say such an integral *converges* if the equivalent iterated integral converges as a succession of one-dimensional integrals. For nonnegative functions, this criterion is the same as most others which are used.

2. Here is one type of improper integral. Suppose $\mathcal{S}$ is the unit square where $0 \le x \le 1$ and $0 \le y \le 1$. Does either of these converge? If not, explain why. If yes, attempt to evaluate.

a) $\iint_{\mathcal{S}} \frac{1}{x+y} \, dA$ b) $\iint_{\mathcal{S}} \frac{1}{xy} \, dA$

3. Here is another type of improper integral. Does either of these converge? If not, explain why. If yes, attempt to evaluate.

a) $\iint_{\mathbb{R}^2} e^{-|x|-|y|} \, dA$ b) $\iint_{\mathbb{R}^2} e^{-|x+y|} \, dA$

4. **Size** Suppose that $f(x,y) = (y - x^2)e^{x^2+y^2}$. Determine which of the integrals I_1, I_2, I_3, and I_4 below is largest, which next largest, etc. Explain carefully how you reached your conclusions. (Hint: where is f positive? Where negative?) Do not attempt to evaluate the integrals explicitly.

$$I_1 = \int_0^2 \int_0^4 f(x,y) \, dy \, dx \qquad I_2 = \int_0^2 \int_0^{x^2} f(x,y) \, dy \, dx$$

$$I_3 = \int_0^2 \int_{x^2}^4 f(x,y) \, dy \, dx \qquad I_4 = \int_0^2 \int_0^4 |f(x,y)| \, dy \, dx$$