640:251:01-03 # Workshop Problems **3/26/97**

1. Consider the double integral

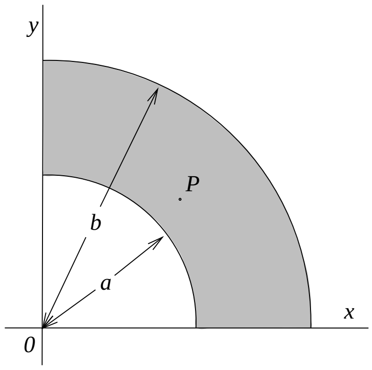
$$\int_0^2 \int_{-x}^{x} (2+x)\,dy\,dx$$

a) Evaluate it directly.

b) Evaluate it by reversing the order of integration (that is, converting to an integral, or sum of integrals, in $dx\,dy$ order).

c) Evaluate it using polar coordinates.

2. a) Suppose that a and b are positive numbers with $a < b$. What are the coordinates of P, the center of mass* of the region made up of the points in the first quadrant whose distance from the origin is between a and b? Note that if the coordinates of P are $(\overline{x}, \overline{y})$, then $\overline{x} = \overline{y}$ since the region is symmetric with respect to the line $y = x$: you need to compute just one coordinate of P.



b) If a region is *convex* (so any line segment whose endpoints are in the region must lie entirely within the region), then the center of mass of the region must be inside the region. But the center of mass of a non-convex region may be *outside* the region. For example, if the region illustrated above has a and b rather "large" compared to a "small" $b - a$, then P may be outside the region. Find specific numbers a and b for which this is true.

c) Now suppose that $a = 1$. Find the value of b which has P sitting exactly on the inner boundary of the region.

Optional: construct a model of the region described in c). Check its properties!

3. Suppose that D is a region in the xy-plane of area A. Find the area of the portion of the plane $ax + by + cz = d$, where $c \neq 0$, which lies above D.

Point out where the assumption $c \neq 0$ is needed in doing the problem and explain geometrically why this is a natural assumption to make. Explain geometrically why your answer depends on d in the way that it does.

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* This is also called the "center of gravity" or (for a homogeneous mass distribution) the "centroid".