1. Suppose $z = \dfrac{x^2y}{y+1}$. Compute $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

Answer $z_x = \dfrac{2xy}{y+1}$, because all the "y" stuff is just a constant when we do this derivative. The other derivative uses the quotient rule: $z_y = \dfrac{(x^2)(y+1)-(1)(x^2y)}{(y+1)^2} = \dfrac{x^2}{(y+1)^2}$, but you *don't* need to "simplify" the answer to receive full credit on this quiz.

2. If $F(x,y) = \dfrac{10x^2+y^2}{x^2-7y^2}$, then:

a) Compute both $\lim_{x\to 0} F(x,0)$ and $\lim_{y\to 0} F(0,y)$.

Answer We see that $F(x,0) = \dfrac{10x^2+0^2}{x^2-(7)0^2} = 10$ (if $x \neq 0$) so the limit is easy to compute: $\lim_{x\to 0} F(x,0) = 10$. Also, $F(0,y) = \dfrac{(10)0^2+y^2}{0^2-7y^2} = -\dfrac{1}{7}$ (if $x \neq 0$) so $\lim_{x\to 0} F(0,y) = -\dfrac{1}{7}$.

b) Does $\lim_{(x,y)\to(0,0)} F(x,y)$ exist? Explain your answer briefly.

Answer No, because $10 \neq -\dfrac{1}{7}$ and the limits along any two approaches to $(0,0)$ must coincide for the two-dimensional limit to exist.

3. If $G(s,t) = te^{s-t}$, compute $\dfrac{\partial^2 G}{\partial s^2} + \dfrac{\partial^2 G}{\partial t^2}$.

Answer The specific combination of partial derivatives which you are asked to compute here occurs in *many* mathematical models of physical phenomena. It has its own name: it is called "the Laplacian". So this problem asks you to compute the Laplacian of G. I don't see any real shortcuts:

$G_s = te^{s-t}$ (because this is just a constant multiplying e^s) and $G_{ss} = te^{s-t}$. The other derivatives need the product rule. $G_t = e^{s-t} + te^{s-t}(-1)$ and $G_{tt} = e^{s-t}(-1) + e^{s-t}(-1) + te^{s-t}(-1)^2 = (t-2)e^{s-t}$. So the Laplacian of G is $(2t-2)e^{s-t}$. The simplification indicated here is, again, *not* required. One notation frequently used for the Laplacian of G is $\triangle G$.

4. Suppose $H(x,y) = f(x^2+y)$ where f is an unknown differentiable function of one variable. Compute $\dfrac{\partial H}{\partial x} - 2x\dfrac{\partial H}{\partial y}$.

Answer You need the chain rule. You can't do anything about f other than indicate its derivative because you don't know further information. So: $H_x = f'(x^2+y)2x$ and $H_y = f'(x^2+y)1$ and assembling the pieces we see that $H_x - 2xH_y = f'(x^2+y)2x - 2xf'(x^2+y) = 0$. Computations like this arise frequently in the solution of certain partial differential equations arising in applications. That this computation does not depend on the choice of the function f is striking and important.