Here are answers that would earn full credit. Other methods may also be valid.

(8) 1. The derivative of $f(x) = \frac{4}{5-x^2}$ is $f'(x) = \frac{8x}{(x^2-5)^2}$. Use this information to write an equation for the line tangent to $y = \frac{4}{5-x^2}$ when $x = 2$.

Answer $f(2) = \frac{4}{5-2^2} = 4$ and $f'(2) = \frac{8\cdot 2}{(2^2-5)^2} = 16$. The line is $y - 4 = 16(x-2)$, a fine answer. If you wish, the answer also can be given as $y = 16x - 28$.

(9) 2. Compute these limits. Give some brief supporting evidence for your answers.

a) $\lim_{x\to 2} \frac{x^2-2x}{x+1}$

Answer Since $2 + 1 \neq 0$ we can use the algebraic rules and just "plug in". The answer is $\frac{2^2-2\cdot 2}{2+1} = 0$.

b) $\lim_{x\to 2} \frac{x^2-2x}{x-2}$

Answer Because $2 - 2 = 0$ we look further. $x^2 - 2x = (x-2)x$ so since $x \neq 2$ (true here since we're investigating $\lim_{x\to 2}$), $\frac{x^2-2x}{x-2} = \frac{(x-2)x}{x-2} = x$. Therefore $\lim_{x\to 2} \frac{x^2-2x}{x-2} = \lim_{x\to 2} x = 2$.

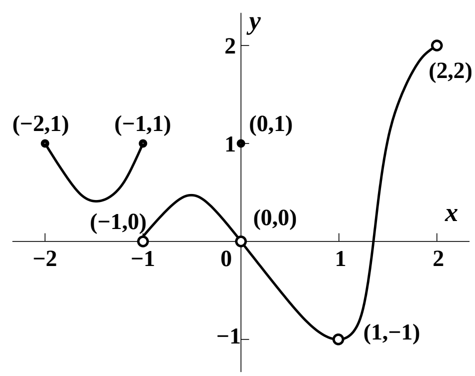
c) $\lim_{x\to 0} \frac{\sin(2x)}{3x}$

Answer $\lim_{x\to 0} \frac{\sin(2x)}{3x} = \lim_{x\to 0} \frac{2}{3}\cdot\frac{\sin(2x)}{2x} = \frac{2}{3}\cdot 1 = \frac{2}{3}$.

(10) 3. Suppose $f(x) = \sqrt{3x+1}$. Use algebraic properties of limits to compute $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

Answer $f(x+h) = \sqrt{3(x+h)+1}$ so that $\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{3(x+h)+1}-\sqrt{3x+1}}{h} = \frac{\sqrt{3(x+h)+1}-\sqrt{3x+1}}{h}\cdot\frac{\sqrt{3(x+h)+1}+\sqrt{3x+1}}{\sqrt{3(x+h)+1}+\sqrt{3x+1}} =$
$\frac{\left(\sqrt{3(x+h)+1}\right)^2-\left(\sqrt{3x+1}\right)^2}{h\left(\sqrt{3(x+h)+1}+\sqrt{3x+1}\right)} = \frac{\left((3(x+h)+1)-(3x+1)\right)}{h\left(\sqrt{3(x+h)+1}+\sqrt{3x+1}\right)} = \frac{(3x+3h+1-3x-1)}{h\left(\sqrt{3(x+h)+1}+\sqrt{3x+1}\right)} = \frac{3h}{h\left(\sqrt{3(x+h)+1}+\sqrt{3x+1}\right)} = \frac{3}{\left(\sqrt{3(x+h)+1}+\sqrt{3x+1}\right)}$.
Wow! This last form has a bottom (denominator) which does not approach 0 as $h \to 0$, and "plugging in" (substituting) will work. So now we know $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3}{\left(\sqrt{3(x+h)+1}+\sqrt{3x+1}\right)} = \frac{3}{2\sqrt{3x+1}}$.

(12) 4. Here is a graph of $y = f(x)$ which should be used to answer the questions following as well as possible.



The entire graph of $y = f(x)$

a) What, exactly, is the domain of $f(x)$? **Answer** $-2 \le x < 1$ and $1 < x < 2$.

What, exactly, is the range of $f(x)$? **Answer** $-1 < y < 2$.

b) Answers to the following questions should either be a specific real number which is the limit value, or **DNE** if the requested limit does not exist.

What is $\lim_{x\to -1^-} f(x)$? **Answer** 1

What is $\lim_{x\to -1^+} f(x)$? **Answer** 0

What is $\lim_{x\to -1} f(x)$? **Answer DNE**

What is $\lim_{x\to 0^-} f(x)$? **Answer** 0

What is $\lim_{x\to 0^+} f(x)$? **Answer** 0

What is $\lim_{x\to 0} f(x)$? **Answer** 0

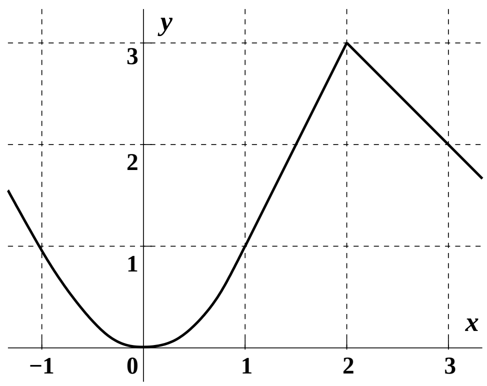
(10) 5. Suppose $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ Ax + B & \text{if } 1 \le x \le 2 \\ 5 - x & \text{if } 2 < x \end{cases}$.

a) Find A and B so that $\lim_{x\to 1} f(x)$ and $\lim_{x\to 2} f(x)$ both exist.

Answer $\lim_{x\to 1^-} f(x)$, the left-hand limit at 1, is $\lim_{x\to 1^-} x^2 = 1$, and $\lim_{x\to 1^+} f(x)$, the right-hand limit at 2, is $\lim_{x\to 1^+} Ax + B = A + B$. So the limit as $x \to 1$ will exist if $1 = A + B$. Similarly, $\lim_{x\to 2^-} f(x)$, the left-hand limit at 2, is $\lim_{x\to 2^-} Ax + B = 2A + B$, and $\lim_{x\to 2^+} f(x)$, the right-hand limit at 2, is $\lim_{x\to 2^+} 5 - x = 3$. So the limit as $x \to 1$ will exist if $2A + B = 3$.

Therefore we must find A and B so that $\begin{cases} A + B = 1 \\ 2A + B = 3 \end{cases}$. If we subtract the second equation from the first, the result is $A = 2$. Then the first equation (or the second) gives $B = -1$. Those are the values requested.

b) Use the values of A and B found in a) to sketch the graph of $y = f(x)$ on the axes given.



(6) 6. What is the natural domain of $f(x) = \sqrt{8-x} + \dfrac{5}{\sqrt{x+4}}$?

Answer $\sqrt{8-x}$ is defined if $8 - x \ge 0$, which is the same as $x \le 8$. $\sqrt{x+4}$ is defined when $x + 4 \ge 0$ but, since $\sqrt{x+4}$ is on the bottom (o.k. again, denominator) it should not be 0, so $x + 4 > 0$. This is $x > -4$. Both of the inequalities must be satisfied, so the natural domain of $f(x)$ is $-4 < x \le 8$.