

Math 135, section F2, summer 2006

From data to derivatives to graphs

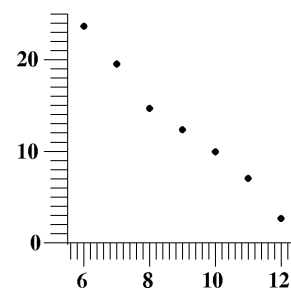
An example

Here is the U.S. inflation rate for the months of June through December in the year 1920 (!). Each number represents an annualized percentage rate of change of the U.S. consumer price index (CPI) as measured by price changes for the month cited.

June	July	August	September	October	November	December
23.67%	19.54%	14.69%	12.36%	9.94%	7.03%	2.65%

Translation

Suppose $C(t)$ is the U.S. CPI for the months listed, and t represents the month of the year (so $t = 6$ represents June). The data points given are shown as a graph to the right.



Use the numbers cited to answer the following questions as well as you can.

1. What is the sign of $C'(t)$ for t in the interval $[6, 12]$?

Answer Remember that the rates quoted represent the *changes* in CPI. Since all of the changes shown are positive, I think that $C'(t) > 0$ for t in $[6, 12]$.

2. What is the sign of $C''(t)$ for t in the interval $[6, 12]$?

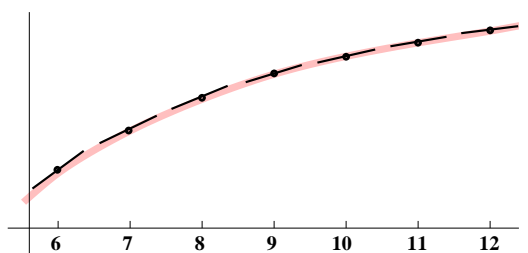
Answer Since the numbers shown are *decreasing*, and the numbers themselves are rates of change (sort of – so they are almost $C'(t)$ numbers) I think that $C''(t) < 0$ for t in the interval $[6, 12]$.

3. Describe in English what is happening to the CPI in the interval $[6, 12]$??

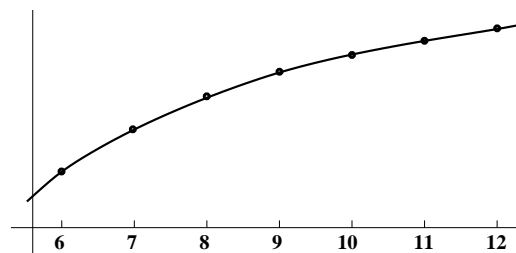
Answer Prices increase throughout the time interval considered, but the rate of increase is slowing.

4. Sketch a qualitatively correct graph of the function $C(t)$ in the interval $[6, 12]$.

Answer



A possible graph of U.S. CPI during part of 1920



A possible graph of U.S. CPI during part of 1920

Here I attempted to sketch a *possible* CPI curve. I wanted the tangent line segments to be qualitatively correct: all tilting up, but the tilts decreasing from left to right.

Here the tangent line segments have been *erased* but the suggested curve has been enhanced (darkened?). The possible CPI points have also been kept.

Such a curve is called concave down.

This is all slightly incorrect. The inflation figures are annualized percentage figures, and therefore are not rates of change but quotients of the CPI at successive times. But the idea is (essentially!) correct.

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1. Here is 1931 inflation data. $C(t)$ is the consumer price index. Describe what you can about $C'(t)$ and $C''(t)$. Describe the behavior of $C(t)$ in words. Draw a plausible and qualitatively correct graph of $C(t)$.

January	February	March	April	May	June
−7.02%	−7.65%	−7.69%	−8.82%	−9.47%	−10.12%

2. Here is 1998 inflation data. $C(t)$ is the consumer price index. Describe what you can about $C'(t)$ and $C''(t)$. Describe the behavior of $C(t)$ in words. Draw a plausible and qualitatively correct graph of $C(t)$.

May	June	July	August	September	October
1.69%	1.68%	1.68%	1.62%	1.49%	1.49%

3. Here is 1965 inflation data. $C(t)$ is the consumer price index. Describe what you can about $C'(t)$ and $C''(t)$. Describe the behavior of $C(t)$ in words. Draw a plausible and qualitatively correct graph of $C(t)$.

January	February	March	April	May	June
0.97%	0.97%	1.29%	1.62%	1.62%	1.94%

4. Salt (NaCl, sodium chloride) has a saturation amount (the most salt that can be dissolved) which varies with temperature. Here is the amount of salt in grams which can be dissolved in 100 mL (milliliters) of water (H_2O) at various temperatures:

°C:	0	10	20	30	40	50
Mass:	35.7	35.8	36.0	36.3	36.6	37.0

Suppose $S(c)$ is the saturation amount as a function of the temperature. Describe what you can about $S'(c)$ and $S''(c)$. Describe the behavior of $S(c)$ in words. Draw a plausible and qualitatively correct graph of $S(c)$.

5. Here are average weekly newspaper readers aged 18–24 in each indicated year as an absolute number (in 1,000's) and as a percentage of the population:

Year	1998	1999	2000	2001	2002	2003	2004	2005
Numbers	7,052	6,881	6,655	6,574	7,107	7,241	7,322	7,287
% of Pop.	43.5	42.1	39.9	38.9	40.9	39.7	39.0	38.4

Suppose $P(y)$ is the percentage of the 18–24 aged population reading newspapers in year y . Describe what you can about $P'(y)$ and $P''(y)$. Describe the behavior of $P(y)$ in words. Draw a plausible and qualitatively correct graph of $P(y)$.

6. Moose population; Isle Royal; explain; relevance to Easter Island and the oil industry.